2-LOCAL ISOMETRIES OF NON-COMMUTATIVE LORENTZ SPACES

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Dedicated to E. I. Gordon on the occasion of his 70th birthday

Abstract. Let $\mathcal{M}$ be a von Neumann algebra equipped with a faithful normal finite trace $\tau$, and let $S(\mathcal{M}, \tau)$ be a $\tau$-algebra of all $\tau$-measurable operators affiliated with $\mathcal{M}$. For $x \in S(\mathcal{M}, \tau)$ the generalized singular value function $\mu(x) : t \mapsto \mu(t; x)$, $t > 0$, is defined by the equality $\mu(t; x) = \inf\{\|xp\|_{\mathcal{M}} : p^2 = p^* = p \in \mathcal{M}, \tau(1 - p) \leq t\}$. Let $\psi$ be an increasing concave continuous function on $[0, \infty)$ with $\psi(0) = 0$, $\psi(\infty) = \infty$, and let $\Lambda_\psi(\mathcal{M}, \tau) = \{x \in S(\mathcal{M}, \tau) : \|x\|_\psi = \int_0^\infty \mu(t; x) d\psi(t) < \infty\}$ be the non-commutative Lorentz space. A surjective (not necessarily linear) mapping $V : \Lambda_\psi(\mathcal{M}, \tau) \to \Lambda_\psi(\mathcal{M}, \tau)$ is called a surjective 2-local isometry, if for any $x, y \in \Lambda_\psi(\mathcal{M}, \tau)$ there exists a surjective linear isometry $V_{x,y} : \Lambda_\psi(\mathcal{M}, \tau) \to \Lambda_\psi(\mathcal{M}, \tau)$ such that $V(x) = V_{x,y}(x)$ and $V(y) = V_{x,y}(y)$. It is proved that in the case when $\mathcal{M}$ is a factor, every surjective 2-local isometry $V : \Lambda_\psi(\mathcal{M}, \tau) \to \Lambda_\psi(\mathcal{M}, \tau)$ is a linear isometry.

Key words: measurable operator, Lorentz space, isometry.

Mathematical Subject Classification (2010): 46L52, 46B04.


1. Introduction

Let $\mathcal{H}$ be a complex separable infinite-dimensional Hilbert space, let $(C_E, \| \cdot \|_{C_E})$ be a Banach ideal of compact linear operators in $\mathcal{H}$ generated by symmetric sequence space $(E, \| \cdot \|_E) \subset c_0$, and let $V$ be a surjective 2-local isometry on $C_E$, that is, $V : C_E \to C_E$ is a surjective (not necessarily linear) mapping such that for any $x, y \in C_E$ there exists a surjective linear isometry $V_{x,y}$ on $C_E$ for which $V(x) = V_{x,y}(x)$ and $V(y) = V_{x,y}(y)$. In the papers [1, 2] it is shown that in the case when $C_E$ is separable or has the Fatou property, $C_E \neq C_{I_2}$, every surjective 2-local isometry on $C_E$ is a linear isometry. In the proof of this statement is essentially used explicit description of all surjective linear isometries on $C_E$ [1, 3].

Banach ideals $(C_E, \| \cdot \|_{C_E})$ of compact linear operators are examples of non-commutative symmetric spaces $E(\mathcal{M}, \tau)$ of measurable operators affiliated with a von Neumann algebra $\mathcal{M}$.
equipped with a faithful normal semifinite trace $\tau$ (see, for example, [4, Ch. 2, § 2.5]). It is natural to expect that for these non-commutative symmetric spaces with the Fatou property, every surjective 2-local isometry $V : E(M, \tau) \to E(M, \tau)$ is a linear map. Unfortunately, the method of proof of a similar statement for Banach ideals $(C_E, \| \cdot \|_{C_E})$ can not be applied here, since there is no description of surjective linear isometries $V : E(M, \tau) \to E(M, \tau)$. At the same time, in the case of non-commutative Lorentz and Marcinkiewicz spaces, such a description of surjective linear isometries was obtained in the paper [5]. Using this description, we obtain the following description of surjective 2-local isometries of non-commutative Lorentz spaces.

**Theorem 1.** Let $M$ be an arbitrary factor with a faithful normal finite trace $\tau$, and let $(\Lambda_\psi(M, \tau), \| \cdot \|_\psi)$ be a non-commutative Lorentz space. Then every surjective 2-local isometry $V : \Lambda_\psi(M, \tau) \to \Lambda_\psi(M, \tau)$ is a linear isometry.

2. Preliminaries

Let $\mathcal{H}$ be an infinite-dimensional complex Hilbert space, let $\mathcal{B}(\mathcal{H})$ be the $C^*$-algebra of all bounded linear operators in $\mathcal{H}$, and let $1$ be the unit in $\mathcal{B}(\mathcal{H})$. Let $M \subseteq \mathcal{B}(\mathcal{H})$ be a von Neumann algebra on Hilbert space $\mathcal{H}$ equipped with a faithful normal semifinite trace $\tau$ (see, for example, [6]). A linear operator $x : \mathcal{D}(x) \to \mathcal{H}$, where the domain $\mathcal{D}(x)$ of $x$ is a linear subspace of $\mathcal{H}$, is said to be affiliated with $M$ if $yx \subseteq xy$ for all $y \in M'$, where $M'$ is the commutant of $M$. A linear operator $x : \mathcal{D}(x) \to \mathcal{H}$ is termed measurable with respect to $M$ if $x$ is closed, densely defined, affiliated with $M$ and there exists a sequence $\{p_n\}_{n=1}^\infty$ in the lattice $\mathcal{P}(M)$ of all projections of $M$, such that $p_n \uparrow 1$, $p_n(\mathcal{H}) \subseteq \mathcal{D}(x)$ and $1 - p_n$ is a finite projection (with respect to $M$) for all $n$. The collection $S(M)$ of all measurable operators with respect to $M$ is a unital $*$-algebra with respect to strong sums and products.

Let $x$ be a self-adjoint operator affiliated with $M$ and let $\{e^\lambda\}$ be a spectral measure of $x$. It is well known that if $x$ is a closed operator affiliated with $M$ with the polar decomposition $x = u|x|$, then $u \in M$ and $e \in M$ for all projections $e \in \{e^|x|\}$. Moreover, $x \in S(M)$ if and only if $x$ is closed, densely defined, affiliated with $M$ and $e^{|x|}(\lambda, \infty)$ is a finite projection for some $\lambda > 0$.

An operator $x \in S(M)$ is called $\tau$-measurable if there exists a sequence $\{p_n\}_{n=1}^\infty$ in $\mathcal{P}(M)$ such that $p_n \uparrow 1$, $p_n(\mathcal{H}) \subseteq \mathcal{D}(x)$ and $\tau(1 - p_n) < \infty$ for all $n$. The collection $S(M, \tau)$ of all $\tau$-measurable operators is a unital $*$-subalgebra of $S(M)$. It is well known that a linear operator $x$ belongs to $S(M, \tau)$ if and only if $x \in S(M)$ and there exists $\lambda = \lambda(x) > 0$ such that $\tau(e^{|x|}(\lambda, \infty)) < \infty$.

The generalized singular value function $\mu(x) : t \to \mu(t; x), t > 0$, of the operator $x \in S(M, \tau)$ is defined by setting [7]

$$\mu(t; x) = \inf \bigg\{ \|xp\| : p \in \mathcal{P}(M), \tau(1 - p) \leq t \bigg\} = \inf \bigg\{ s > 0 : \tau(e^{|x|}(s, \infty)) \leq t \bigg\}.$$  

A non-zero linear subspace $E(M, \tau) \subseteq S(M, \tau)$ with the Banach norm $\| \cdot \|_{E(M, \tau)}$ is called a symmetric space if the conditions

$$x \in E(M, \tau), \quad y \in S(M, \tau), \quad \mu_t(y) \leq \mu_t(x) \quad \text{for all} \quad t > 0,$$

imply that $y \in E(M, \tau)$ and $\|y\|_{E(M, \tau)} \leq \|x\|_{E(M, \tau)}$.

It is known that in the case $\tau(1) < \infty$ it is true

$$S(M) = S(M, \tau) \quad \text{and} \quad M \subseteq E(M, \tau) \subseteq L_1(M, \tau).$$
for each symmetric space $E(\mathcal{M}, \tau)$, where

$$L_1(\mathcal{M}, \tau) = \left\{ x \in S(\mathcal{M}, \tau) : \|x\|_1 = \int_0^\infty \mu_t(x) \, dt < \infty \right\}.$$  

In addition,

$$\mathcal{M} \cdot E(\mathcal{M}, \tau) \cdot \mathcal{M} \subseteq E(\mathcal{M}, \tau),$$

and

$$\|axb\|_{E(\mathcal{M}, \tau)} \leq \|a\|_{\mathcal{M}} \cdot \|b\|_{\mathcal{M}} \cdot \|x\|_{E(\mathcal{M}, \tau)}$$

for all $a, b \in \mathcal{M}, x \in E(\mathcal{M}, \tau)$.

Let $\psi$ be an increasing concave continuous function on $[0, \infty)$ with $\psi(0) = 0, \psi(\infty) = \lim_{t \to \infty} \psi(t) = \infty$, and let

$$\Lambda_\psi(\mathcal{M}, \tau) = \left\{ x \in S(\mathcal{M}, \tau) : \|x\|_\psi = \int_0^\infty \mu_t(x) \, d\psi(t) < \infty \right\}$$

be the non-commutative Lorentz space. It is known that $(\Lambda_\psi(\mathcal{M}, \tau), \| \cdot \|_\psi)$ is a symmetric space [8], and the norm $\| \cdot \|_\psi$ has the Fatou property, that is, the conditions $0 \leq x_k \in \Lambda_\psi(\mathcal{M}, \tau)$ for all $k$, and $\sup_{k \geq 1} \|x_k\|_\psi < \infty$, imply that there exists $0 \leq x \in \Lambda_\psi(\mathcal{M}, \tau)$ such that $x_k \uparrow x$ and $\|x\|_\psi = \sup_{k \geq 1} \|x_k\|_\psi$.

Denote by $M_\psi(\mathcal{M}, \tau)$ the set of all $x \in S(\mathcal{M}, \tau)$ for which

$$\|x\|_{M_\psi} = \sup_{t > 0} \frac{1}{\psi(t)} \int_0^t \mu(s; x) \, ds$$

is finite. The set $M_\psi(\mathcal{M}, \tau)$ with the norm $\| \cdot \|_{M_\psi}$ is a symmetric space which is called a Marcinkiewicz space.

Denote by $M_\psi^0(\mathcal{M}, \tau)$ the closure of $\mathcal{M}$ in $M_\psi(\mathcal{M}, \tau)$. It is known [9] that the conjugate space of $(\Lambda_\psi(\mathcal{M}, \tau), \| \cdot \|_\psi)$ is identified with $(M_\psi(\mathcal{M}, \tau), \| \cdot \|_{M_\psi})$, and the conjugate space of $(M_\psi^0(\mathcal{M}, \tau), \| \cdot \|_{M_\psi})$, under the condition $\lim_{t \to 0} \frac{1}{\psi(t)} = 0$, is identified with $(\Lambda_\psi(\mathcal{M}, \tau), \| \cdot \|_\psi)$. The duality in these pairs of spaces is realized via the bilinear form $(x, y) = \tau(xy)$. It should be pointed out that the spaces $(\Lambda_\psi(\mathcal{M}, \tau), \| \cdot \|_\psi), (M_\psi(\mathcal{M}, \tau), \| \cdot \|_{M_\psi})$ and $(M_\psi^0(\mathcal{M}, \tau), \| \cdot \|_{M_\psi})$ are symmetric spaces [4, Ch. 2, § 2.6], [8].

### 3. Isometries of Non-Commutative Lorentz Spaces

Let $\mathcal{M} \subseteq B(\mathcal{H})$ be a von Neumann algebra on Hilbert space $\mathcal{H}$. A linear bijective mapping $\Phi : \mathcal{M} \to \mathcal{M}$ is called a Jordan isomorphism if $\Phi(x^2) = (\Phi(x))^2$ and $\Phi(x^*) = (\Phi(x))^*$ for all $x \in \mathcal{M}$.

If $\Phi : \mathcal{M} \to \mathcal{M}$ is a Jordan isomorphism, then there exists a central projection $z \in \mathcal{M}$ such that $\Phi_+(x) = \Phi(x) \cdot z, x \in \mathcal{M}$, is an $*$-homomorphism, and $\Phi_-(x) = \Phi(x) \cdot (1 - z), x \in \mathcal{M}$, is an $*$-antihomomorphism (see, for example, [10, Ch. 3, § 3.2.1]). Consequently, if $\mathcal{M}$ is a factor then a Jordan isomorphism $\Phi : \mathcal{M} \to \mathcal{M}$ is an $*$-homomorphism or $*$-antihomomorphism.

If $\tau$ is a faithful normal finite trace on von Neumann algebra $\mathcal{M}$ then a Jordan isomorphism $\Phi : \mathcal{M} \to \mathcal{M}$ is continuous with respect to measure topology $t_\tau$ generated by trace $\tau$ (see, for
example, [11, Ch. 5, § 3, Proposition 1]). Therefore, \( \Phi \) extends to a \( t_{\tau} \)-continuous Jordan isomorphism \( \tilde{\Phi} : S(\mathcal{M}, \tau) \to S(\mathcal{M}, \tau) \). In addition, if \( \tau(\Phi(x)) = \tau(x) \) for all \( x \in \mathcal{M} \) then \( \mu(t; \Phi(x)) = \mu(t; x) \) for all \( t \in S(\mathcal{M}, \tau) \), in particular, \( \Phi(E(\mathcal{M}, \tau)) = E(\mathcal{M}, \tau) \) and \( \| \Phi(x) \|_{E(\mathcal{M}, \tau)} = \| x \|_{E(\mathcal{M}, \tau)} \) for all \( x \in E(\mathcal{M}, \tau) \), that is, \( \Phi : E(\mathcal{M}, \tau) \to E(\mathcal{M}, \tau) \) is a surjective linear isometry for any symmetric space \( (E(\mathcal{M}, \tau), \| \cdot \|_{E(\mathcal{M}, \tau)}) \).

Thus, it is true the following

**Proposition 1.** Let \( \mathcal{M} \) be an arbitrary von Neumann algebra with a faithful normal finite trace \( \tau \), and let \( \Phi : \mathcal{M} \to \mathcal{M} \) be a Jordan isomorphism such that \( \tau(\Phi(x)) = \tau(x) \) for all \( x \in \mathcal{M} \). Then for every symmetric space \( (E(\mathcal{M}, \tau), \| \cdot \|_{E(\mathcal{M}, \tau)}) \) the mapping \( \tilde{\Phi} : E(\mathcal{M}, \tau) \to E(\mathcal{M}, \tau) \) given by the equality \( \tilde{\Phi}(x) = u \cdot \tilde{\Phi}(x) \cdot v, x \in E(\mathcal{M}, \tau), u, v \) are unitary operators in \( \mathcal{M} \), is a surjective linear isometry.

We need the following description of surjective linear isometries of the spaces \((\Lambda_{\psi}(\mathcal{M}, \tau), \| \cdot \|_{\psi})\) and \((M_{\psi}^{0}(\mathcal{M}, \tau), \| \cdot \|_{M_{\psi}})\) [5, Theorems 5.1, 6.1].

**Theorem 2.** Let \( \mathcal{M} \) be an arbitrary von Neumann algebra with a faithful normal finite trace \( \tau \), and let \( V : \Lambda_{\psi}(\mathcal{M}, \tau) \to \Lambda_{\psi}(\mathcal{M}, \tau) \) (respectively, \( V : M_{\psi}^{0}(\mathcal{M}, \tau) \to M_{\psi}^{0}(\mathcal{M}, \tau) \)) be a surjective linear isometry. Then there exist uniquely an unitary operator \( u \in \mathcal{M} \) and a Jordan isomorphism \( \Phi : \mathcal{M} \to \mathcal{M} \) such that \( V(x) = u \cdot \Phi(x) \) and \( \tau(\Phi(x)) = \tau(x) \) for all \( x \in \mathcal{M} \).

4. Local Isometries of Non-Commutative Lorentz Spaces

Let \((X, \| \cdot \|_{X})\) be an arbitrary Banach space over the field \( \mathbb{K} \) of complex or real numbers. A surjective (not necessarily linear) mapping \( T : X \to X \) is called a surjective 2-local isometry [2], if for any \( x, y \in X \) there exists a surjective linear isometry \( V_{x,y} : X \to X \) such that \( T(x) = V_{x,y}(x) \) and \( T(y) = V_{x,y}(y) \). It is clear that every surjective linear isometry on \( X \) is a surjective 2-local isometry on \( X \). In addition,

\[
T(\lambda x) = V_{x,\lambda x}(\lambda x) = \lambda V_{x,\lambda x}(x) = \lambda T(x)
\]

for any \( x \in X \) and \( \lambda \in \mathbb{K} \).

Consequently, in order to establish linearity of a 2-local isometry \( T \), it is sufficient to show that \( T(x + y) = T(x) + T(y) \) for all \( x, y \in X \).

Since

\[
\| T(x) - T(y) \|_{X} = \| V_{x,y}(x) - V_{x,y}(y) \|_{X} = \| x - y \|_{X}
\]

for all \( x, y \in X \), it follows that \( T \) is continuous map on \((X, \| \cdot \|_{X})\). In addition, in the case a real Banach space \( X (\mathbb{K} = \mathbb{R}) \), every surjective 2-local isometry on \( X \) is a linear map (see Mazur–Ulam Theorem [12, Ch. 1, § 1.3, Theorem 1.3.5]). In the case a complex Banach space \( X (\mathbb{K} = \mathbb{C}) \), this fact is not true.

Using the description of all surjective linear isometries on a separable Banach symmetric ideal \( \mathcal{C}_{E} [3] \) (respectively, on a Banach symmetric ideal \( \mathcal{C}_{E} \) with Fatou property [1]), \( \mathcal{C}_{E} \neq \mathcal{C}_{l_{2}} \), in the papers [1, 2] it is proved that every surjective 2-local isometry \( T : \mathcal{C}_{E} \to \mathcal{C}_{E} \) is a linear isometry.

The following Theorem is a version of the above results for the spaces \( \Lambda_{\psi}(\mathcal{M}, \tau) \) and \( M_{\psi}^{0}(\mathcal{M}, \tau) \).

**Theorem 3.** Let \( \mathcal{M} \) be an arbitrary factor with a faithful normal finite trace \( \tau \), and let \( T : \Lambda_{\psi}(\mathcal{M}, \tau) \to \Lambda_{\psi}(\mathcal{M}, \tau) \) (respectively, \( T : M_{\psi}^{0}(\mathcal{M}, \tau) \to M_{\psi}^{0}(\mathcal{M}, \tau) \)) be a surjective 2-local isometry. Then \( T \) is a linear isometry.
Fix $x, y \in \mathcal{M}$ and let $V_{x,y} : \Lambda_\psi(\mathcal{M}, \tau) \to \Lambda_\psi(\mathcal{M}, \tau)$ be a surjective isometry such that $T(x) = V_{x,y}(x)$ and $T(y) = V_{x,y}(y)$. By Theorem 2, there exist uniquely an unitary operator $u \in \mathcal{M}$ and a Jordan isomorphism $\Phi : \mathcal{M} \to \mathcal{M}$ such that $V_{x,y}(a) = u \cdot \Phi(a)$ and $\tau(\Phi(a)) = \tau(a)$ for all $a \in \mathcal{M}$. Since $\mathcal{M}$ is a factor it follows then $\Phi : \mathcal{M} \to \mathcal{M}$ is an $*$-isomorphism or $\Phi$ is an $*$-anti-isomorphism.

We assume that $\Phi$ is an $*$-isomorphism (in the case when $\Phi$ is an $*$-anti-isomorphism, the proof is similar).

We have

$$\tau(T(x) \cdot (T(y))^*) = \tau(V_{x,y}(x) \cdot (V_{x,y}(y))^*)$$

$$= \tau(u \cdot \Phi(x) \cdot (u \cdot \Phi(y))^*) = \tau(u \cdot \Phi(xy^*) \cdot u^*) = \tau(\Phi(xy^*)) = \tau(xy^*).$$

Consequently, $\tau(T(x) \cdot (T(y))^*) = \tau(xy^*)$ for all $x, y \in \mathcal{M}$.

If $x, y, z \in \mathcal{M}$, then

$$\tau(T(x + y) \cdot (T(z))^*) = \tau((x + y)z^*), \quad \tau(T(x) \cdot T(z)^*) = \tau(xz^*),$$

$$\tau(T(y) \cdot T(z)^*) = \tau(y \cdot z^*).$$

Therefore

$$\tau((T(x + y) - T(x) - T(y)) \cdot (T(z))^*) = 0$$

for all $z \in \mathcal{M}$. Taking $z = x + y$, $z = x$ and $z = y$, we obtain

$$\tau((T(x + y) - T(x) - T(y)) \cdot ((T(x + y) - T(x) - T(y))^*) = 0,$n

that is, $T(x + y) = T(x) + T(y)$ for all $x, y \in \mathcal{M}$.

Since the Lorentz space $\Lambda_\psi(0, \infty)$ of measurable functions on a semi-axis $[0, \infty)$ is separable space [13, Ch. 2I, §5], it follows that the non-commutative Lorentz $(\Lambda_\psi(\mathcal{M}, \tau), \| \cdot \|_\psi)$ has an order continuous norm [14, Proposition 3.6], that is, $\|x_n\|_\psi \downarrow 0$ whenever $x_n \in \Lambda_\psi(\mathcal{M}, \tau)$ and $x_n \downarrow 0$. Consequently, the factor $\mathcal{M}$ is dense in the space $\Lambda_\psi(\mathcal{M}, \tau)$. Since $T$ is a continuous mapping on $\Lambda_\psi(\mathcal{M}, \tau)$ it follows that $T(x + y) = T(x) + T(y)$ for all $x, y \in \Lambda_\psi(\mathcal{M}, \tau)$, that is, $T$ is a surjective linear isometry.

For the space $M^0_\psi(\mathcal{M}, \tau)$, the proof of the linearity of the surjective 2-local isometry $T : M^0_\psi(\mathcal{M}, \tau) \to M^0_\psi(\mathcal{M}, \tau)$ repeats the previous proof. ⊥

References


2-ЛОКАЛЬНЫЕ ИЗОМЕТРИИ НЕКОММУТАТИВНЫХ ПРОСТРАНСТВ ЛОРЕНЦА

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Аннотация. Пусть $\mathcal{M}$ алгебра фон Неймана с точным нормальным конечным следом $\tau$, и пусть $S(\mathcal{M}, \tau)$ инволютивная алгебра всех $\tau$-измеримых операторов, присоединенных к алгебре $\mathcal{M}$. Для оператора $x \in S(\mathcal{M}, \tau)$ невозрастающая вероятностная выборка $\mu(x) : t \to \mu(t; x)$, $t > 0$, определяется с помощью равенства $\mu(t; x) = \inf\{\|xp\|_{\mathcal{M}} : p^2 = p = p \in \mathcal{M}, \tau(1 - p) \leq t\}$. Пусть $\psi$ возрастающая вогнутая непрерывная функция на $[0, \infty)$, для которой $\psi(0) = 0$, $\psi(\infty) = \infty$. Пусть $\Lambda_{\psi}(\mathcal{M}, \tau) = \{x \in S(\mathcal{M}, \tau) : \|x\|_{\psi} = \int_0^{\infty} \mu(t; x)d\psi(t) < \infty\}$ некоммутативное пространство Лоренца. Сюръективное (не обязательно линейное) отображение $V : \Lambda_{\psi}(\mathcal{M}, \tau) \to \Lambda_{\psi}(\mathcal{M}, \tau)$ называется сюръективной 2-локальной изометрией, если для любых $x, y \in \Lambda_{\psi}(\mathcal{M}, \tau)$ существует такая сюръективная линейная изометрия $V_{x,y} : \Lambda_{\psi}(\mathcal{M}, \tau) \to \Lambda_{\psi}(\mathcal{M}, \tau)$, что $V(x) = V_{x,y}(x)$ и $V(y) = V_{x,y}(y)$. Доказано, что в случае, когда $\mathcal{M}$ есть фактор, каждая сюръективная 2-локальная изометрия $V : \Lambda_{\psi}(\mathcal{M}, \tau) \to \Lambda_{\psi}(\mathcal{M}, \tau)$ есть линейная изометрия.

Ключевые слова: измеримый оператор, пространство Лоренца, изометрия.

Mathematical Subject Classification (2010): 46L52, 46B04.