Vladikavkaz Mathematical Journal
2022, Volume 24, Issue 1, P. 24–35

УДК 517.951
DOI 10.46698/d1853-7650-4105-n

A NONEXISTENCE RESULT FOR THE SEMI-LINEAR MOORE–GIBSON–THOMPSON EQUATION WITH NONLINEAR MEMORY ON THE HEISENBERG GROUP

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Abstract. The Moore–Gibson–Thompson theory was developed starting from a third order differential equation, built in the context of some consideration related fluid mechanics. Subsequently the equation was considered as a heat conduction equation because it has been obtained by considering a relaxation parameter into the type III heat conduction. Since the advent of the Moore–Gibson–Thompson theory, the number of dedicated studies to this theory has increased considerably. The Moore–Gibson–Thompson equation modifies and defines equations for thermal conduction and mass diffusion that occur in solids. In this paper we investigate a class of Moore–Gibson–Thompson equation with nonlinear memory on the Heisenberg group.The problem of nonexistence of global weak solutions in the Heisenberg group has received specific attention in the recent years. In the present paper we use the method of test functions to prove nonexistence of global weak solutions. The results obtained in this paper extend several contributions and we focus on new nonexistence results which are due to the presence of the fractional Laplacian operator of order $\frac{\sigma}{2}$.

Key words: Moore–Gibson–Thompson equation, nonlocal operators, Heisenberg group, nonlinear memory.

AMS Subject Classification: 35A01, 35R03, 35R11, 35B33, 35L25.

For citation: *Georgiev, S. G. and Hakem, A.* A Nonexistence Result for the Semi-Linear Moore–Gibson–Thompson Equation with Nonlinear Memory on the Heisenberg Group, *Vladikavkaz Math. J.*, 2022, vol. 24, no. 1, pp. 24–35. DOI: 10.46698/d1853-7650-4105-n.

1. Introduction and Preliminaries

The main goal of this paper is concerned with the nonexistence of global weak solutions for the following semi-linear Moore–Gibson–Thompson equation with nonlinear mixed damping term

$$u_{ttt} + u_{tt} + (-\Delta_{\mathbb{H}})^m u - (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_t = \int\limits_0^t (t-s)^{-\gamma} |u(s)|^p \, ds \tag{1.1}$$

subject to the following initial conditions

$$u(\eta, 0) = u_0(\eta), \quad u_t(\eta, 0) = u_1(\eta), \quad u_{tt}(\eta, 0) = u_2(\eta), \quad \eta \in \mathbb{R}^{2n+1}, \tag{1.2}$$

where $0 < \gamma < 1$, $\sigma \in (0, 2]$, $m \geqslant 1$, $p > 1$ and $\Delta_{\mathbb{H}}$ is the Kohn–Laplace operator on the $(2n + 1)$-dimensional Heisenberg group. The operator $(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}$ accounts for anomalous diffusion. In the paper by W. Chen and A. Palmieri [1], it is investigated the blow-up of the solutions for the following semi linear Cauchy problem for MGT equation in the conservative case with nonlinearity of derivative type

$$\begin{cases} \beta u_{ttt} + u_{tt} - \Delta u - \beta \Delta u_t = |u_t|^p, & x \in \mathbb{R}^n,\ t > 0, \\ (u, u_t, u_{tt})(x, 0) = \varepsilon(u_0, u_1, u_2)(x), & x \in \mathbb{R}^n, \end{cases}$$

where $p > 1$ and ε is a positive parameter describing the size of the initial data. More precisely, they proved that there exists a positive constant ε_0 such that for any $\varepsilon \in (0, \varepsilon_0]$ the solution u blows up in finite time. Furthermore, the upper bound estimate for the lifespan

$$T(\varepsilon) \leqslant \begin{cases} C\varepsilon^{-(\frac{1}{p-1} - \frac{n-1}{2})^{-1}}, & 1 < p < p_{Gl_a}(n), \\ e^{C\varepsilon^{-(p-1)}}, & p = p_{Gl_a}(n), \end{cases}$$

holds, where $C > 0$ is an independent of ε constant and $p_{Gl_a}(n) = \frac{n+1}{n-1}$ is the so called Glassey exponent. The MGT was previously analyzed by several authors from a different point of view. For instance, see the papers [1–4] and references therein for a variety of problems related to this equations. Recently, T. Dao and A. Z. Fino in [5] have proved blow-up results to determine the critical exponents for the following Cauchy problem for semi-linear structurally damped wave model with nonlinear memory

$$\begin{cases} u_{tt} - \Delta u + \mu(-\Delta)^{\frac{\sigma}{2}} u_t = \int\limits_0^t (t-s)^{-\gamma} |u(s)|^p \, ds, & x \in \mathbb{R}^n,\ t > 0\ , \\ u(x, 0) = u_0(x), & u_t(x, 0) = u_1(x), \end{cases}$$

where $\mu > 0$, $\sigma \in (0, 2)$ for some $\gamma \in (0, 1)$ and $p > 1$. Using a modified test function method, if

$$p \leqslant p_c = 1 + \frac{2 + (1-\gamma)(2-\tilde{\sigma})}{\max\left[n - 2 + \gamma(2 - \tilde{\sigma}), 0\right]}, \qquad \int\limits_{\mathbb{R}^{2n+1}} \Big(u_1(x) + (-\Delta)^{\frac{\sigma}{2}}\Big) u_0(x)\, dx > 0,$$

it was shown that there is no global (in time) weak solution. The problem of nonexistence of global solutions in the Heisenberg group has received specific attention in recent years. For instance, see the papers [6–9] and references therein. For more details on Heisenberg groups and partial differential equations in Heisenberg groups, we refer the reader to [7–11] and the references therein.

Motivated by above papers, we investigate the problem (1.1), (1.2) for nonexistence of global weak solutions by using the method of the test function. Our main result is as follows.

Theorem 1.1. *Let* $0 < \sigma \leqslant 2$, $\tilde{\sigma} = \min(\sigma, 1)$ *and* $n \geqslant 1$. *We assume that the initial data* $(u_0, u_1, u_2) \in H^{\sigma}(\mathbb{R}^{2n+1}) \times H^2(\mathbb{R}^{2n+1}) \times \mathbb{L}^2(\mathbb{R}^{2n+1})$ *satisfy the following relation*

$$\int\limits_{\mathbb{R}^{2n+1}} \Big(u_1(\eta) + u_2(\eta) + (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_0(\eta)\Big)\, d\eta > 0. \tag{1.3}$$

If

$$p \leqslant p_c = \frac{Q + 2 - \tilde{\sigma}}{(2 - \tilde{\sigma})\gamma + Q - 2m}, \tag{1.4}$$

or

$$p < \frac{1}{\gamma},$$

then there exists no global nontrivial weak solution to (1.1)–(1.2).

The paper is organized as follows. In the next section, we give some auxiliary results. In Section 3 we prove our main result.

2. Auxiliary Results

For the sake of the reader, in this section we give some known facts about the Heisenberg group $\mathbb{H}$ and the operator $\Delta_{\mathbb{H}}$.

The Heisenberg group $\mathbb{H}$ whose points will be denoted by $\eta = (x, y, \tau)$, is the Lie group $(\mathbb{R}^{2n+1}; \circ)$ with the non-commutative group operation $\circ$ defined by

$$\eta \circ \eta' = (x + x', y + y', \tau + \tau' + 2\,(x \cdot y' - x' \cdot y))$$

for all $\eta = (x, y, \tau)$, $\eta' = (x', y', \tau') \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$, where $\cdot$ denotes the standard inner product in $\mathbb{R}^n$. This group operation endows $\mathbb{H}$ with the structure of a Lie group. The Laplacian $\Delta_{\mathbb{H}}$ over $\mathbb{H}$ is obtained from the vector fields $X_i = \frac{\partial}{\partial x_i} + 2y_i \frac{\partial}{\partial \tau}$, $Y_i = \frac{\partial}{\partial y_i} - 2x_i \frac{\partial}{\partial \tau}$ and we have

$$\Delta_{\mathbb{H}} = \sum_{i=1}^{n} \left(X_i^2 + Y_i^2 \right).$$

Observe that the vector field $T = \frac{\partial}{\partial \tau}$ does not appear in the equality above. This fact makes us presume a "loss of derivative" in the variable τ. The compensation comes from the relation

$$[X_i, Y_j] = -4T, \quad i, j \in 1, 2, 3, \dots, n.$$

Then the Heisenberg group $\mathbb{H}$ is a nilpotent Lie group of order 2. Explicit computation gives the expression

$$\Delta_{\mathbb{H}} = \sum_{i=1}^{n} \left(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + 4y_i \frac{\partial^2}{\partial x_i \partial \tau} - 4x_i \frac{\partial^2}{\partial y_i \partial \tau} + 4\left(x_i^2 + y_i^2\right) \frac{\partial^2}{\partial \tau^2} \right).$$

A natural group of dilatations on $\mathbb{H}$ is given by

$$\delta_\lambda(\eta) = \left(\lambda x, \lambda y, \lambda^2 \tau\right), \quad \lambda > 0,$$

whose Jacobian determinant is λ^Q, where $Q = 2n + 2$ is the homogeneous dimension of $\mathbb{H}$. The operator $\Delta_{\mathbb{H}}$ is a degenerate elliptic operator. It is invariant with respect to the left translation of $\mathbb{H}$ and homogeneous with respect to the dilations δ_λ. More precisely, we have

$$\Delta_{\mathbb{H}}(u(\eta \circ \eta')) = (\Delta_{\mathbb{H}} u)(\eta \circ \eta'), \quad \Delta_{\mathbb{H}}(u \circ \delta_\lambda) = \lambda^2 (\Delta_{\mathbb{H}} u) \circ \delta_\lambda, \quad \eta, \eta' \in \mathbb{H}.$$

The natural distance from η to the origin is introduced by Folland and Stein, see [10],

$$|\eta|_{\mathbb{H}} = \left(\tau^2 + \left(\sum_{i=1}^{n} \left(x_i^2 + y_i^2 \right) \right)^2 \right)^{\frac{1}{4}}.$$

Now, we will collect some preliminary knowledge that will be used hereafter.

Definition 2.1 [12]. A function $f : [a, b] \to \mathbb{R}$ is said to be absolutely continuous if there exists a Lebesgue summable function $\varphi \in \mathbb{L}^1(a, b)$ such that

$$f(t) = f(a) + \int_a^t \varphi(s)\, ds, \quad t \in [a, b].$$

We denote by $AC[0, T]$ the space of all functions which are absolutely continuous on $[0, T]$ with $0 < T < \infty$.

DEFINITION 2.2 [12]. Let $f \in \mathbb{L}^1(0,T)$ with $T > 0$. The Riemann–Liouville left- and right-sided fractional integrals of order $\alpha \in (0,1)$ are defined by

$$I^{\alpha}_{0|t} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{-(1-\alpha)} f(s)\, ds, \quad t > 0, \tag{2.1}$$

and

$$I^{\alpha}_{t|T} f(t) = \frac{1}{\Gamma(\alpha)} \int_t^T (t-s)^{-(1-\alpha)} f(s)\, ds, \quad t < T, \tag{2.2}$$

respectively, where Γ is the Euler gamma function.

DEFINITION 2.3 [12]. Let $f \in AC[0,T]$ with $T > 0$. The Riemann–Liouville left- and right-sided fractional derivatives of order $\in \alpha \in (0,1)$ are defined by

$$D^{\alpha}_{0|t} f(t) = \frac{d}{dt} I^{1-\alpha}_{0|t} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-s)^{-\alpha} f(s)\, ds, \quad t > 0, \tag{2.3}$$

$$D^{\alpha}_{t|T} f(t) = -\frac{d}{dt} I^{1-\alpha}_{t|T} f(t) = -\frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_t^T (t-s)^{-\alpha} f(s)\, ds, \quad t < T, \tag{2.4}$$

respectively.

Proposition 2.1 [12]. *Let $T > 0$ and $\alpha \in (0,1)$. The fractional integration by parts formula*

$$\int_0^T f(t) D^{\alpha}_{0|t}\, g(t)\, dt = \int_0^T g(t) D^{\alpha}_{t|T}\, f(t)\, dt, \tag{2.5}$$

is valid for every $f \in I^{\alpha}_{t|T}(\mathbb{L}^p(0,T))$ and $g \in I^{\alpha}_{0|t}(\mathbb{L}^q(0,T))$ such that $\frac{1}{p} + \frac{1}{q} \leqslant 1 + \alpha$ with $p, q > 1$, where

$$I^{\alpha}_{t|T}(\mathbb{L}^p(0,T)) = \left\{ f = I^{\alpha}_{t|T} h,\ h \in \mathbb{L}^p(0,T) \right\}$$

and

$$I^{\alpha}_{0|t}(\mathbb{L}^q(0,T)) = \left\{ f = I^{\alpha}_{0|t} h,\ h \in \mathbb{L}^q(0,T) \right\}.$$

Proposition 2.2 [13]. *Let $T > 0$ and $\alpha \in (0,1)$. Then, we have the following identities*

$$D^{\alpha}_{0|t} I^{\alpha}_{0|t} f(t) = f(t), \quad \text{a.e.} \quad t \in (0,T) \quad \text{for all} \quad f \in \mathbb{L}^r(0,T) \quad \text{with} \quad 1 \leqslant r \leqslant \infty, \tag{2.6}$$

and

$$(-1)^m D^m D^{\alpha}_{t|T} f = D^{m+\alpha}_{t|t} f \quad \text{for all} \quad f \in AC^{m+1}[0,T], \tag{2.7}$$

where

$$AC^{m+1}[0;T] = \left\{ f : [0,T] \longrightarrow \mathbb{R}, \quad \text{such that} \quad D^m f \in AC[0,T] \right\}$$

and $D^m = \frac{d^m}{dt^m}$ is the usual m times derivative.

Lemma 2.1 [13]. *Let $T > 0$, $0 < \alpha < 1$ and $m \geqslant 0$. For all $t \in [0,T]$, we have*

$$D^{m+\alpha}_{t|T} \left(1 - \frac{t}{T}\right)^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-m-\alpha)} T^{-(m+\alpha)} \left(1 - \frac{t}{T}\right)^{\beta-\alpha-m}. \tag{2.8}$$

Lemma 2.2 [5]**.** *Let $T>0$, $0<\alpha<1$, $m\geqslant 0$ and $p>1$. Then, we have*

$$\int_0^T \psi(t)^{-\frac{1}{p-1}}\left|D^{m+\alpha}_{t|T}\psi(t)\right|^{\frac{p}{p-1}}dt = CT^{1-(m+\alpha)\frac{p}{p-1}}. \tag{2.9}$$

Before to proceed with the proof of our main result, we will give a definition for weak solutions for (1.1)–(1.2).

Definition 2.4. A function $u\in\mathbb{L}^p\left((0,T),\mathbb{L}^p(\mathbb{R}^{2n+1})\right)\cap\mathbb{L}^1\left((0,T),\mathbb{L}^2(\mathbb{R}^{2n+1})\right)$ is called a local weak solution of (1.1)–(1.2) subject to the initial data $(u_0,u_1,u_2)\in H^\sigma(\mathbb{R}^{2n+1})\times H^2(\mathbb{R}^{2n+1})\times\mathbb{L}^2(\mathbb{R}^{2n+1})$ if the following equality

$$\begin{aligned}
\Gamma(\alpha)\int_0^T\int_{\mathbb{R}^{2n+1}} & I^\alpha_{0|t}(|u(\eta,t)|^p)\varphi(\eta,t)\,d\eta\,dt + \int_{\mathbb{R}^{2n+1}}\left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0\right)\varphi(\eta,0)\,d\eta \\
&- \int_{\mathbb{R}^{2n+1}}(u_0(\eta)+u_1(\eta))\varphi_t(\eta,0)\,d\eta + \int_{\mathbb{R}^{2n+1}}u_0(\eta)\varphi_{tt}(\eta,0)\,d\eta \\
&= -\int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)\varphi_{ttt}(\eta,t)\,d\eta\,dt + \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)\varphi_{tt}(\eta,t)\,d\eta\,dt \\
&- \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)(-\Delta_{\mathbb{H}})^m\varphi(\eta,t)\,d\eta\,dt + \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}\varphi_t(\eta,t)\,d\eta\,dt,
\end{aligned}$$

holds for any regular function

$$\varphi\in C^1\left((0,T];H^\sigma\left(\mathbb{R}^{2n+1}\right)\right)\cap C\left((0,T];H^2\left(\mathbb{R}^{2n+1}\right)\right)\cap C^2\left((0,T];\mathbb{L}^2\left(\mathbb{R}^{2n+1}\right)\right),$$

such that $\varphi(\eta,T)=0$, $\varphi_t(\eta,T)=0$ and $\varphi_{tt}(\eta,T)=0$ for all $\eta\in\mathbb{R}^{2n+1}$. The solution u is called global if $T=+\infty$.

3. Proof of the Main Result

Throughout this section, with C we will denote a positive constant whose value may change from line to line. The proof of our main result is based on a contradiction. Suppose that u is a global weak solution to (1.1)–(1.2). Then u satisfies the following equation

$$\begin{aligned}
\Gamma(\alpha)\int_0^T\int_{\mathbb{R}^{2n+1}} & I^\alpha_{0|t}(|u(\eta,t)|^p)\varphi(\eta,t)\,d\eta\,dt + \int_{\mathbb{R}^{2n+1}}\left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0\right)\varphi(\eta,0)\,d\eta \\
&- \int_{\mathbb{R}^{2n+1}}(u_0(\eta)+u_1(\eta))\varphi_t(\eta,0)\,d\eta + \int_{\mathbb{R}^{2n+1}}u_0(\eta)\varphi_{tt}(\eta,0)\,d\eta \\
&= -\int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)\varphi_{ttt}(\eta,t)\,d\eta\,dt + \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)\varphi_{tt}(\eta,t)\,d\eta\,dt \\
&- \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)(-\Delta_{\mathbb{H}})^m\varphi(\eta,t)\,d\eta\,dt + \int_0^T\int_{\mathbb{R}^{2n+1}}u(\eta,t)(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}\varphi_t(\eta,t)\,d\eta\,dt,
\end{aligned} \tag{3.1}$$

where $\alpha = 1-\gamma \in (0,1)$, for all test function φ such that $\varphi(\eta,T) = \varphi_t(\eta,T) = \varphi_{tt}(\eta,T) = 0$ and for all $T >> 1$. We define the following auxiliary functions

$$\phi_R(\eta) = \phi\left(\frac{\eta}{R}\right), \quad \tilde{\varphi}(\eta,t) = \phi_R(\eta)\psi(t), \quad \psi(t) = \left(1-\frac{t}{T}\right)^{\beta},$$

where ϕ is a non-negative smooth function such that

$$\phi(x) = \phi(|x|), \quad \phi(0) = 1, \quad 0 < \phi(r) \leqslant 1, \quad \text{for} \quad r \geqslant 0. \tag{3.2}$$

Moreover, ϕ is decreasing and $\phi(r) \to 0$, as $r \to \infty$ sufficiently fast. Then, we define the test function as follows

$$\varphi(\eta,t) = D^{\alpha}_{t|T}\tilde{\varphi}(\eta,t) = \phi_R(\eta)D^{\alpha}_{t|T}(\psi(t)).$$

From (3.1), using (2.7) and (2.8), we have

$$\begin{aligned}
&\Gamma(\alpha)\int_0^T\int_{\mathbb{R}^{2n+1}} I^{\alpha}_{0|t}(|u(\eta,t)|^p)\, D^{\alpha}_{t|T}\tilde{\varphi}(\eta,t)\,d\eta\,dt - CT^{-1-\alpha}\int_{\mathbb{R}^{2n+1}}(u_0(\eta)+u_1(\eta))\phi_R(\eta)\,d\eta\\
&+CT^{-\alpha}\int_{\mathbb{R}^{2n+1}}\left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0\right)\phi_R(\eta)\,d\eta + CT^{-2-\alpha}\int_{\mathbb{R}^{2n+1}}u_0(\eta)\phi_R(\eta)\,d\eta\\
&=\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)\phi_R(\eta)D^{2+\alpha}_{t|T}(\psi(t))\,d\eta\,dt - \int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)\phi_R(\eta)D^{3+\alpha}_{t|T}(\psi(t))\,d\eta\,dt\\
&\qquad -\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)D^{\alpha}_{t|T}(\psi(t))(-\Delta_{\mathbb{H}})^m\phi_R(\eta)\,d\eta\,dt\\
&\qquad +\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)D^{\alpha}_{t|T}(\psi(t))(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}(\phi_R(\eta))\,d\eta\,dt.
\end{aligned} \tag{3.3}$$

Using (2.5) and then (2.6), we arrive at

$$\begin{aligned}
&I_1 + CT^{-\alpha}\int_{\mathbb{R}^{2n+1}}\left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0\right)\phi_R(\eta)\,d\eta\\
&\quad - CT^{-1-\alpha}\int_{\mathbb{R}^{2n+1}}(u_0(\eta)+u_1(\eta))\phi_R(\eta)\,d\eta + CT^{-2-\alpha}\int_{\mathbb{R}^{2n+1}}u_0(\eta)\phi_R(\eta)\,d\eta\\
&= C\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)\phi_R(\eta)D^{2+\alpha}_{t|T}(\psi(t))\,d\eta\,dt - C\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)\phi_R(\eta)D^{3+\alpha}_{t|T}(\psi(t))\,d\eta\,dt\\
&\qquad - C\int_0^T\int_{\{|\eta|_{\mathbb{H}}\geqslant R\}} u(\eta,t)D^{\alpha}_{t|T}(\psi(t))(-\Delta_{\mathbb{H}})^m\phi_R(\eta)\,d\eta\,dt\\
&+C\int_0^T\int_{\mathbb{R}^{2n+1}} u(\eta,t)D^{\alpha}_{t|T}(\psi(t))(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}(\phi_R(\eta))\,d\eta\,dt = \mathscr{A}(\psi)+\mathscr{B}(\psi)+\mathscr{C}(\psi)+\mathscr{D}(\psi).
\end{aligned} \tag{3.4}$$

Here

$$I_1 = \int_0^T \int_{\mathbb{R}^{2n+1}} |u(\eta,t)|^p \, \tilde{\varphi}(\eta,t)\, d\eta\, dt.$$

On the other hand, using Hölder's inequality with $\frac{1}{p} + \frac{1}{p'} = 1$, where p' is the conjugate of p, we can proceed with the estimate for $\mathscr{A}(\psi)$ as follows

$$\begin{aligned}|\mathscr{A}(\psi)| &\leqslant C \int_0^T \int_{\mathbb{R}^{2n+1}} |u(\eta,t)| \phi_R(\eta) \left| D^{2+\alpha}_{t|T}(\psi(t)) \right| d\eta\, dt \\ &= C \int_0^T \int_{\mathbb{R}^{2n+1}} |u(\eta,t)| \tilde{\varphi}^{\frac{1}{p}}(\eta,t) \tilde{\varphi}^{-\frac{1}{p}}(\eta,t) \phi_R(\eta) \left| D^{2+\alpha}_{t|T}(\psi(t)) \right| d\eta\, dt \\ &\leqslant C I_1^{\frac{1}{p}} \left(\int_0^T \int_{\mathbb{R}^{2n+1}} \phi_R(\eta) (\psi(t))^{-\frac{p'}{p}} \left| D^{2+\alpha}_{t|T}(\psi(t)) \right|^{p'} d\eta\, dt \right)^{\frac{1}{p'}}.\end{aligned}$$

At this stage, we pass to the scaled variables

$$\tilde{t} = \frac{t}{T} \quad \text{and} \quad \tilde{\eta} = (\tilde{x},\ \tilde{y}, \tilde{\tau}),$$

such that

$$\tilde{\tau} = \frac{\tau}{R^2},\ \tilde{x} = \frac{x}{R},\ \tilde{y} = \frac{y}{R}.$$

Using Lemma 2.2, one has

$$|\mathscr{A}(\psi)| \leqslant C I_1^{\frac{1}{p}} R^{\frac{Q}{p'}} T^{\frac{1}{p'} - 2 - \alpha}. \tag{3.5}$$

Similarly, we obtain

$$\begin{aligned}|\mathscr{C}(\psi)| &\leqslant C I_2^{\frac{1}{p}} \left(\int_0^T \int_{\{|\eta|_{\mathbb{H}} \geqslant R\}} (\phi_R(\eta))^{-\frac{p'}{p}} (\psi(t))^{-\frac{p'}{p}} \left| D^{\alpha}_{t|T}(\psi(t)) \right|^{p'} \right. \\ &\qquad \left. \times \left| (-\Delta_{\mathbb{H}})^m \phi_R(\eta) \right|^{p'} d\eta\, dt \right)^{\frac{1}{p'}} \leqslant C I_2^{\frac{1}{p}} R^{\frac{Q}{p'} - 2m} T^{\frac{1}{p'} - \alpha},\end{aligned} \tag{3.6}$$

where

$$I_2 = \int_0^T \int_{\{|\eta|_{\mathbb{H}} \geqslant R\}} |u(\eta,t)|^p \tilde{\varphi}(\eta,t)\, d\eta\, dt.$$

Now, we can proceed with the estimate for $\mathscr{D}(\psi)$ and $\mathscr{B}(\psi)$ in the following manner

$$\begin{aligned}|\mathscr{D}(\psi)| &\leqslant C I_1^{\frac{1}{p}} \left(\int_0^T \int_{\mathbb{R}^{2n+1}} (\phi_R(\eta))^{-\frac{p'}{p}} (\psi(t))^{-\frac{p'}{p}} \left| D^{1+\alpha}_{t|T}(\psi(t)) \right|^{p'} \right. \\ &\qquad \left. \times \left| (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} \phi_R(\eta) \right|^{p'} d\eta\, dt \right)^{\frac{1}{p'}} \leqslant C I_1^{\frac{1}{p}} R^{\frac{Q}{p'} - \sigma} T^{\frac{1}{p'} - \alpha - 1}\end{aligned} \tag{3.7}$$

and

$$\begin{aligned} |\mathscr{B}(\psi)| &\leqslant C \int_0^T \int_{\mathbb{R}^{2n+1}} |u(\eta,t)|\phi_R(\eta) \left| D_{t|T}^{3+\alpha}(\psi(t)) \right| \, d\eta \, dt \\ &= C \int_0^T \int_{\mathbb{R}^{2n+1}} |u(\eta,t)|\tilde{\varphi}^{\frac{1}{p}}(\eta,t)\tilde{\varphi}^{-\frac{1}{p}}(\eta,t)\phi_R(\eta) \left| D_{t|T}^{3+\alpha}(\psi(t)) \right| \, d\eta \, dt \\ &\leqslant C I_1^{\frac{1}{p}} \left(\int_0^T \int_{\mathbb{R}^{2n+1}} \phi_R(\eta)(\psi(t))^{-\frac{p'}{p}} \left| D_{t|T}^{3+\alpha}(\psi(t)) \right|^{p'} \, d\eta \, dt \right)^{\frac{1}{p'}} \leqslant C I_1^{\frac{1}{p}} R^{\frac{Q}{p'}} T^{\frac{1}{p'}-3-\alpha}. \end{aligned} \tag{3.8}$$

Combining the estimates (3.5)–(3.8) into (3.4), one deduces that

$$\begin{aligned} &I_1 + CT^{-\alpha} \int_{\mathbb{R}^{2n+1}} \left(u_1(\eta) + u_2(\eta) + (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_0 \right) \phi_R(\eta) \, d\eta \\ &- CT^{-1-\alpha} \int_{\mathbb{R}^{2n+1}} (u_0(\eta) + u_1(\eta))\phi_R(\eta) \, d\eta + CT^{-2-\alpha} \int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta) \, d\eta \\ &\leqslant C I_1^{\frac{1}{p}} \left(R^{\frac{Q}{p'}-\sigma} T^{\frac{1}{p'}-\alpha-1} + R^{\frac{Q}{p'}} T^{\frac{1}{p'}-2-\alpha} + R^{\frac{Q}{p'}} T^{\frac{1}{p'}-3-\alpha} \right) + C I_2^{\frac{1}{p}} R^{\frac{Q}{p'}-2m} T^{\frac{1}{p'}-\alpha}. \end{aligned} \tag{3.9}$$

Hence, we get

$$\begin{aligned} &I_1 + CT^{-\alpha} \int_{\mathbb{R}^{2n+1}} \left(u_1(\eta) + u_2(\eta) + (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_0(\eta) \right) \phi_R(\eta) \, d\eta \\ &\leqslant C I_1^{\frac{1}{p}} R^{\frac{Q}{p'}} T^{\frac{1}{p'}-\alpha} \left(T^{-2} + T^{-1}R^{-\sigma} + T^{-3} \right) + C I_2^{\frac{1}{p}} R^{\frac{Q}{p'}-2m} T^{\frac{1}{p'}-\alpha} \\ &+ CT^{-1-\alpha} \int_{\mathbb{R}^{2n+1}} (u_0(\eta) + u_1(\eta))\phi_R(\eta) \, d\eta - CT^{-2-\alpha} \int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta) \, d\eta. \end{aligned} \tag{3.10}$$

Because (1.3) holds and $\phi_R(\eta) \to 1$, as $R \to \infty$, there exists a sufficiently large constant $R_0 > 0$ such that we have

$$\int_{\mathbb{R}^{2n+1}} \left(u_1(\eta) + u_2(\eta) + (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_0(\eta) \right) \phi_R(\eta) \, d\eta > 0, \tag{3.11}$$

for all $R > R_0$. It is clear that the inequality (1.4) is equivalent to $1 - \alpha p' + \frac{Q}{2-\tilde{\sigma}} - \frac{2p'm}{2-\tilde{\sigma}} \leqslant 0$. So, we have to consider the following two cases.

Case 1. If $1-\alpha p'+\frac{Q}{2-\tilde{\sigma}}-\frac{2p'm}{2-\tilde{\sigma}} < 0$, then we take $R = T^{\frac{1}{2-\tilde{\sigma}}}$. Hence, using (3.10) and (3.11), we have that

$$\begin{aligned} I_1 \leqslant\; & C I_1^{\frac{1}{p}} T^{\frac{1}{p'}-\alpha+\frac{Q}{(2-\tilde{\sigma})p'}-\frac{2m}{2-\tilde{\sigma}}} \\ &+ C \left(T^{-1-\alpha} - T^{-2-\alpha} \right) \int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta) \, d\eta + CT^{-1-\alpha} \int_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta) \, d\eta. \end{aligned}$$

Thanks to the Young inequality for

$$a = I_1^{\frac{1}{p}}, \quad b = T^{\frac{1}{p'}-\alpha+\frac{Q}{(2-\tilde{\sigma})p'}-\frac{2m}{2-\tilde{\sigma}}},$$

we conclude that

$$\frac{1}{p'}I_1 \leqslant \frac{C}{p'}T^{1-\alpha p'+\frac{Q}{2-\tilde{\sigma}}-\frac{2p'm}{2-\tilde{\sigma}}} + C\left(T^{-1-\alpha}-T^{-2-\alpha}\right)\int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta + CT^{-1-\alpha}\int_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta. \quad (3.12)$$

If $1-\alpha p'+\frac{Q}{2-\tilde{\sigma}}-\frac{2p'm}{2-\tilde{\sigma}}<0$, then by letting $T\to+\infty$, we deduce that $u\equiv 0$. By invoking (3.10), we obtain

$$\int_{\mathbb{R}^{2n+1}} \left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0(\eta)\right)\phi_R(\eta)\,d\eta \leqslant CT^{-1}\int_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta + C\left(T^{-1}-T^{-2}\right)\int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta.$$

Hence, passing to the limit in the above inequality as $T\to+\infty$, one obtains a contradiction with (1.3).

Case 2. If $1-\alpha p'+\frac{Q}{2-\tilde{\sigma}}-\frac{2p'm}{2-\tilde{\sigma}}=0$, then using (3.12), we obtain

$$\frac{1}{p'}I_1 \leqslant \frac{C}{p'} + C\left(T^{-1-\alpha}-T^{-2-\alpha}\right)\int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta + CT^{-1-\alpha}\int_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta.$$

Hence, it follows that $I_1\leqslant C$, as $T\to+\infty$. By the dominated convergence theorem, one has

$$\int_0^{+\infty}\int_{\mathbb{R}^{2n+1}} |u(\eta,t)|^p\,d\eta\,dt = \lim_{T\to+\infty}\int_0^{T}\int_{\mathbb{R}^{2n+1}} |u(\eta,t)|^p\tilde{\varphi}(\eta,t)\,d\eta\,dt = \lim_{T\to+\infty} I_1 \leqslant C,$$

which yields $u\in\mathbb{L}^p\left((0,+\infty)\times\mathbb{R}^{2n+1}\right)$. On the other hand, repeating the same calculations as above with $R=T^{\frac{1}{2-\tilde{\sigma}}}L^{-\frac{1}{2-\tilde{\sigma}}}$, where $1\leqslant L<R$ is large enough such that when $R\to+\infty$ we do not have $L\to+\infty$ at the same time, we arrive at the following inequality

$$I_1 \leqslant CI_1^{\frac{1}{p}}\left(T^{-\frac{2(1-\tilde{\sigma})}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}} + T^{-\frac{4-3\tilde{\sigma}}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}} + T^{-\frac{\sigma-\tilde{\sigma}}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}+\frac{2}{2-\tilde{\sigma}}}\right)$$
$$+ I_2^{\frac{1}{p}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}+\frac{2m}{2-\tilde{\sigma}}} + C\left(T^{-1-\alpha}-T^{-2-\alpha}\right)\int_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta + CT^{-1-\alpha}\int_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta.$$

Applying the Young inequality for

$$\begin{cases} a=I_1^{\frac{1}{p}}, & b=T^{-\frac{2(1-\tilde{\sigma})}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}},\\ a=I_1^{\frac{1}{p}}, & b=T^{-\frac{\sigma-\tilde{\sigma}}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}+\frac{2}{2-\tilde{\sigma}}},\\ a=I_1^{\frac{1}{p}}, & b=T^{-\frac{4-3\tilde{\sigma}}{2-\tilde{\sigma}}}L^{-\frac{Q}{(2-\tilde{\sigma})p'}}, \end{cases}$$

one concludes that

$$\frac{1}{p'}I_1 \leqslant \frac{C}{p'}\Big(T^{-\frac{2(1-\tilde\sigma)p'}{2-\tilde\sigma}}L^{-\frac{Q}{2-\tilde\sigma}} + T^{-\frac{(4-3\tilde\sigma)p'}{2-\tilde\sigma}}L^{-\frac{Q}{(2-\tilde\sigma)}} + T^{-\frac{(\sigma-\tilde\sigma)p'}{2-\tilde\sigma}}L^{-\frac{Q-2p'}{2-\tilde\sigma}}\Big) + I_2^{\frac{1}{p}}L^{-\frac{Q}{(2-\tilde\sigma)p'}+\frac{2m}{2-\tilde\sigma}} + C\left(T^{-1-\alpha}-T^{-2-\alpha}\right) \times \int\limits_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta + CT^{-1-\alpha}\int\limits_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta. \quad (3.13)$$

We have to distinguish the following two cases:

• If $\sigma \in (0,1]$, then $\sigma = \tilde\sigma$. Consequently, using the fact that $u \in \mathbb{L}^p\left((0,+\infty)\times\mathbb{R}^{2n+1}\right)$, one has

$$\lim_{T\to+\infty} I_2 = \lim_{T\to+\infty} \int\limits_{\{|\eta|_{\mathbb{H}}\geqslant T^{\frac{1}{2-\sigma}}L^{-\frac{1}{2-\sigma}}\}} |u(\eta,t)|^p\tilde\varphi(\eta,t)\,d\eta\,dt = 0.$$

Taking into account the inequality (3.13), by letting $T\to+\infty$, it follows that

$$\int\limits_0^{+\infty}\int\limits_{\mathbb{R}^{2n+1}} |u(\eta,t)|^p\,d\eta\,dt \leqslant C\left(L^{-\frac{Q}{2-\sigma}} + L^{-\frac{Q-2p'}{2-\sigma}}\right).$$

Applying similar arguments as in Case 1, one concludes the desired result.

• If $\sigma\in(1,2]$, then $\tilde\sigma = 1$. Due to the fact that $u \in \mathbb{L}^p\left((0,+\infty)\times\mathbb{R}^{2n+1}\right)$, we get

$$\lim_{T\to+\infty} I_2 = \lim_{T\to+\infty} \int\limits_0^T \int\limits_{\{|\eta|_{\mathbb{H}}\geqslant TL^{-1}\}} |u(\eta,t)|^p\tilde\varphi(\eta,t)\,d\eta\,dt = 0,$$

which implies, as $T\to+\infty$,

$$\int\limits_0^{+\infty}\int\limits_{\mathbb{R}^{2n+1}} |u(\eta,t)|^p\,d\eta\,dt \leqslant CL^{-Q}.$$

Employing similar arguments as in Case 1, one obtains a contradiction with the fact that

$$\int\limits_{\mathbb{R}^{2n+1}} \left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0(\eta)\right)d\eta > 0.$$

Case 3. If $p < \frac{1}{\gamma}$. Substituting $R = \log T$ in (3.1), we derive

$$\begin{aligned} &I_1 + CT^{-\alpha}\int\limits_{\mathbb{R}^{2n+1}} \left(u_1(\eta)+u_2(\eta)+(-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}}u_0(\eta)\right)\phi_R(\eta)\,d\eta \\ &\leqslant CI_1^{\frac{1}{p}}(\log(T))^{\frac{Q}{p'}}T^{\frac{1}{p'}-\alpha}\left(T^{-2}+T^{-1}R^{-\sigma}+T^{-3}\right) + CI_2^{\frac{1}{p}}(\log(T))^{\frac{Q}{p'}-2}T^{\frac{1}{p'}-\alpha} \quad (3.14)\\ &+ C\left(T^{-1-\alpha}-T^{-2-\alpha}\right)\int\limits_{\mathbb{R}^{2n+1}} u_0(\eta)\phi_R(\eta)\,d\eta + CT^{-1-\alpha}\int\limits_{\mathbb{R}^{2n+1}} u_1(\eta)\phi_R(\eta)\,d\eta. \end{aligned}$$

Letting $R \to +\infty$ in the above inequality, we obtain

$$\int\limits_{\mathbb{R}^{2n+1}} \Big(u_1(\eta) + u_2(\eta) + (-\Delta_{\mathbb{H}})^{\frac{\sigma}{2}} u_0(\eta)\Big)\, d\eta \leqslant 0,$$

where we have used the fact that $\frac{1}{p'} - \alpha < 0$. This is the desired contradiction and this completes the proof of our main result.

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Received July 17, 2021

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Владикавказский математический журнал
2022, Том 24, Выпуск 1, С. 24–35

ОТСУТСТВИЕ РЕШЕНИЯ У ПОЛУЛИНЕЙНОГО УРАВНЕНИЯ МУРА — ГИБСОНА — ТОМСОНА С НЕЛИНЕЙНОЙ ПАМЯТЬЮ НА ГРУППЕ ГЕЙЗЕНБЕРГА

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Аннотация. Теория Мура — Гибсона — Томпсона была разработана, начиная с дифференциального уравнения третьего порядка, на основе некоторых соображений, связанных с механикой жидкости. Впоследствии это уравнение рассматривалось как уравнение теплопроводности, поскольку оно было получено путем учета параметра релаксации в теплопроводности типа III. С момента появления теории Мура — Гибсона — Томпсона значительно возросло количество исследований, посвященных этой теории. Уравнение Мура — Гибсона — Томпсона изменяет и определяет уравнения теплопроводности и диффузии массы, возникающие в твердых телах. В этой статье мы исследуем класс уравнений Мура — Гибсона — Томпсона с нелинейной памятью на группе Гейзенберга. Проблеме отсутствия глобальных слабых решений на группе Гейзенберга в последние годы привлекает внимание исследователей. В настоящей работе мы используем метод тестовых функций для доказательства отсутствия глобальных слабых решений. Полученные результаты расширяют несколько предшествующих достижений, причем особое внимание уделяется эффекту несуществования решения, обусловленному наличием оператора Лапласа дробного порядка.

Ключевые слова: уравнение Мура — Гибсона — Томпсона, нелокальный оператор, группа Гейзенберга, нелинейная память.

AMS Subject Classification: 35A01, 35R03, 35R11, 35B33, 35L25.

Образец цитирования: *Georgiev, S. G. and Hakem, A.* A Nonexistence Result for the Semi-Linear Moore–Gibson–Thompson Equation with Nonlinear Memory on the Heisenberg Group // Владикавк. мат. журн.—2022.—Т. 24, № 1.—С. 24–35 (in English). DOI: 10.46698/d1853-7650-4105-n.