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ON EXTREME EXTENSION OF POSITIVE OPERATORS[1]

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To Professor Georgiĭ Georgievich Magaril–Il'yaev
in occasion of his 80th birthday

Abstract. Given vector lattices E, F and a positive operator S from a majorzing subspace D of E to F, denote by $\mathscr{E}(S)$ the collection of all positive extensions of S to all of E. This note aims to describe the collection of extreme points of the convex set $\mathscr{E}(T\circ S)$. It is proved, in particular, that $\mathscr{E}(T\circ S)$ and $T\circ\mathscr{E}(S)$ coincide and every extreme point of $\mathscr{E}(T\circ S)$ is an extreme point of $T\circ\mathscr{E}(S)$, whenever $T:F\to G$ is a Maharam operator between Dedekind complete vector lattices. The proofs of the main results are based on the three ingredients: a characterization of extreme points of subdifferentials, abstract disintegration in Kantorovich spaces, and an intrinsic characterization of subdifferentials.

Keywords: vector lattice, positive operator, extreme extension, subdifferential, Maharam operator.

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1. Introduction

Let E and F be vector lattices and D a vector subspace of E. Given a positive operator $S:D\to F$, denote by $\mathscr{E}(S)$ the collection of all positive extensions of S to all of E; in symbols,

$$\mathscr{E}(S):=\big\{R\in L(E,F):\ R\geqslant 0 \text{ and } R|_G=S\big\},$$

where $L(E,F)$ stands for the vector space of all linear operators from E to F and an operator means a linear map between two vector spaces. Denote by $\operatorname{ext}\mathscr{E}(S)$ the collection of *extreme points* of $\mathscr{E}(S)$, i.e., $R\in\operatorname{ext}\mathscr{E}(S)$ if and only if for any two positive extensions R_1,R_2 of S the equation $R=\alpha_1R_1+\alpha_2R_2$ with $0<\alpha_1,\alpha_2\in\mathbb{R}$, $\alpha_1+\alpha_2=1$, implies $R=R_1=R_2$.

Kantorovich classical result on the extension of positive operators amounts to saying that $\mathscr{E}(S)\neq\varnothing$ whenever F is Dedekind complete and D *majorizes* E, that is, for each $x\in E$ there exists some $y\in D$ with $x\leqslant y$, see [1, Theorem 1.32]. Under the same assumptions, Z. Lipecki, D. Plachky, and W. Thomsen [2, Theorem 1] established that the convex set $\mathscr{E}(T)$

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also has extreme points, that is, $\operatorname{ext}\mathscr{E}(S) \neq \varnothing$. A more general result, stating that the convex set $\mathscr{E}(S)$ (and in fact any support set) not only has extreme points, but also can be recovered from its o-extreme points, was obtained by S. S. Kutateladze [3, Theorem 1], see also [4, p. 98]. An intrinsic characterization of support sets in order-topological terms was obtained by A. G. Kusraev and S. S. Kutateladze [5, Theorems 1–4].

This note aims to identify under what conditions on an operator $T : F \to G$ the equality $\mathscr{E}(T \circ S) = T \circ \mathscr{E}(S)$ and the inclusion $\operatorname{ext}\mathscr{E}(T \circ S) \subset T \circ \operatorname{ext}\mathscr{E}(S)$ occur. The study has been motivated by the author's article [6] on disintegration in order complete vector lattices and the Lipecki's memoir [7] on the set of quasi-measure extensions of a given quasi-measure.

We refer to Aliprantis and Burkinshaw [1] for the needed information from the theory of positive operators. All vector lattices are assumed to be real and Archimedean.

2. The Results

Some definitions are needed to formulate the main results. An operator $T : F \to G$ between vector lattices is said to be *interval preserving* whenever $T([0, x]) = [0, Tx]$ for all $x \in F_+$ and *order continuous* if $\inf_\alpha Tx_\alpha = 0$ in G for every decreasing net (x_α) in F with $\inf_\alpha x_\alpha = 0$, see [1, Definition 1.53 and Theorem 1.56]. Evidently, an interval preserving operator is positive. A *Maharam operator* is an order continuous interval preserving operator, see [9, 4.4.1]. Say that T is *strictly positive* whenever $T(|x|) = 0$ implies $x = 0$.

An operator $P : X \to F$ is said to be *sublinear* whenever P is *subadditive* and *positively homogeneous*, i. e., $P(x + y) \leqslant P(x) + P(y)$ $(x, y \in X)$ and $P(\lambda x) = \lambda P(x)$ $(0 \leqslant \lambda \in \mathbb{R}, x \in X)$, respectively. The *support set* or a *subdifferential at zero* ∂P of a sublinear operator P is the collection of all linear operators from X to F dominated by P:

$$\partial P := \big\{S \in L(X, F) : \ (\forall\, x \in X)\ Sx \leqslant P(x)\big\}.$$

Now the main result of this note can be stated as follows.

Theorem 1. *Let X be a vector space, whilst F and G are Dedekind complete vector lattices. Assume that $P : X \to F$ is a sublinear operator and $T : F \to G$ is a Maharam operator. Then the following inclusion holds:*

$$\operatorname{ext}\partial(T \circ P) \subset T \circ \operatorname{ext}\partial(P).$$

Moreover, if $T \circ R \in \operatorname{ext}\partial(T \circ P)$ for some $R \in \partial P$, then necessarily $R \in \operatorname{ext}\partial(P)$.

The following two results on extreme extensions of positive operators can be deduced from Theorem 1. Below we assume that D, E, F and G are vector lattices with F and G Dedekind complete and D a majorizing sublattice of E.

Theorem 2. *Let $S : D \to F$ be a positive operator and $T : F \to G$ a Maharam operator. Then the following relations hold:*

$$\mathscr{E}(T \circ S) = T \circ \mathscr{E}(S);$$
$$\operatorname{ext}\mathscr{E}(T \circ S) \subset T \circ \operatorname{ext}\mathscr{E}(S).$$

Moreover, if $T \circ R \in \operatorname{ext}\mathscr{E}(T \circ S)$ for some positive operator $R : E \to F$, then $R \in \operatorname{ext}\mathscr{E}(S)$.

Denote by $\Lambda := \operatorname{Orth}(G)$ the f-algebra of all orthomorphisms on G (see Definitions 2.41 and 2.53, Theorems 2.43 and 2.59 in [1]). If $T : F \to G$ is a strictly positive Maharam operator then F can be equipped with the structure of a Λ-module in such a way that F becomes a module homomorphism, so that $T(\sum_{i=1}^{n}\lambda_i u_i) = \sum_{i=1}^{n}\lambda_i T(u_i)S$ for all $\lambda_1,\dots,\lambda_n \in \Lambda$ and $u_1,\dots,u_n \in F$, see [9, Theorem 4.4.3].

Fix a nonempty set A and denote by $\mathscr{P}_{\mathrm{fin(A)}}$ the collection of all finite subsets of A. Assume that a family $(S_\alpha)_{\alpha\in\mathrm{A}}$ of positive operators $S_\alpha : D \to G$ is point-wise order summable, i. e., the net $\big(\sum_{\alpha\in\theta} S_\alpha(x)\big)_{\theta\in\mathscr{P}_{\mathrm{fin(A)}}}$ is order convergent for all $x \in D$. Then we can define the positive operator $S : D \to G$ by $Sx := o\text{-}\sum_{\alpha\in\mathrm{A}} S_\alpha(x) := o\text{-}\lim_{\theta\in\mathscr{P}_{\mathrm{fin(A)}}} \sum_{\alpha\in\theta} S_\alpha(x)$ $(x \in D)$.

Theorem 3. *Every family $(\hat{S}_\alpha)_{\alpha\in\mathrm{A}}$ with $\hat{S}_\alpha \in \mathscr{E}(S_\alpha)$ is point-wise order summable and the formula $\hat{S}x := o\text{-}\sum_{\alpha\in\mathrm{A}} \hat{S}_\alpha(x)$, $x \in E$ defines a member of $\mathscr{E}(S)$. Moreover, if the mapping Σ from $\prod_{\alpha\in\mathrm{A}} \mathscr{E}(S_\alpha)$ to $\mathscr{E}(S)$ is defined as $(\hat{S}_\alpha) \mapsto \hat{S}$ then the following hold:*

$$\mathscr{E}(S) = \Sigma\bigg(\prod_{\alpha\in\mathrm{A}} \mathscr{E}(S_\alpha)\bigg);$$

$$\Sigma^{-1}\big(\operatorname{ext}\mathscr{E}(S)\big) \subset \prod_{\alpha\in\mathrm{A}} \operatorname{ext}\mathscr{E}(S_\alpha).$$

Remark 1. A very special case of Theorem 3 is the following fact obtained by Z. Lipecki [7, Theorema 6.1]: If $\mu, \mu_\alpha : \mathscr{B} \to \mathbb{R}$ are positive finitely additive measures on some algebra of sets, $\mu(B) := \Sigma(\mu_\alpha) : B \mapsto \sum_\alpha \mu_\alpha(B)$ for all $B \in \mathscr{B}$, and $\mathscr{E}(\mu)$ stands for the collection of all extensions of μ to a larger algebra $\widehat{\mathscr{B}}$ preserving positivity and finite additivity, then

$$\mathscr{E}(\mu) = \Sigma\bigg(\prod_{\alpha\in\mathrm{A}} \mathscr{E}(\mu_\alpha)\bigg),$$

$$\Sigma^{-1}\big(\operatorname{ext}\mathscr{E}(\mu)\big) \subset \prod_{\alpha\in\mathrm{A}} \operatorname{ext}\mathscr{E}(\mu_\alpha),$$

where Σ is the operator from $\prod_{\alpha\in\mathrm{A}} \mathscr{E}(\mu_\alpha)$ to $\mathscr{E}(\mu)$ sending $(\hat{\mu}_\alpha)$ to $\Sigma(\hat{\mu}_\alpha)$ with $\hat{\mu}_\alpha$ being an extension to $\widehat{\mathscr{B}}$ of μ_α.

3. Auxiliaries

For the proofs we need some auxiliary results. First, we consider an operator version of the well-known Strassen disintegration theorem. Clearly, $T \circ \partial P \subset \partial(T \circ P)$ for every positive operator T; however, the converse is true only under additional conditions on T.

Theorem 4. *Let F and G be Dedekind complete vector lattice and let T be a Maharam operator from F into G. Then, for an arbitrary sublinear operator P from any vector space X to F, the representation holds*

$$\partial(T \circ P) = T \circ \partial P.$$

$\lhd$ This is an abstract disintegration result obtained by A. G. Kusraev in [6]; see also [9, § 4.4 and § 4.5] for more details on disintegration in vector lattices. $\rhd$

Theorem 5. *Assume that $T : F \to G$ is linear, $P : X \to F$ is sublinear, and $R \in \partial P$. Then $T \circ R$ is an extreme point of $\partial(T \circ P)$ if and only if for any $x \in X$, $y \in F$ we have:*

$$Ty^+ = \inf\big\{T\big((P(u) - Ru) \vee (P(u-x) - R(u-x) + y)\big) : \ u \in X\big\}.$$

$\lhd$ This result was obtained by S. S. Kutateladze in [3]; see also [5, Theorem 2.2.5]. $\rhd$

REMARK 2. Essentially, Theorem 5 generalizes the characterization of extreme points in the scalar case ($F = G = \mathbb{R}$) known as the *Buck-Phelps theorem*, see Holmes [8, 13**D**].

A net (S_i) in Ω is said to be *point-wise o-convergent* to $S \in L(X, F)$ if the net $(S_i x)$ is o-convergent to Sx in F for all $x \in X$. Denote by $o\text{-}\mathrm{cl}(\Omega)$ the collection of all operators S that are the limits of point-wise o-convergent nets in Ω. Say that Ω is *point-wise o-closed* whenever $\Omega = o\text{-}\mathrm{cl}(\Omega)$. The vector lattice of all orthomorphisms on E is denoted by $\mathrm{Orth}(E)$. *The operator convex hull* (or Λ-*convex hull*) $\mathrm{co}_\Lambda(\Omega)$ of a set $\Omega \subset L(X, F)$ is defined as

$$\mathrm{co}_\Lambda(\Omega) = \left\{ \sum_{i=1}^{m} \lambda_i S_i : \ S_1, \dots, S_k \in \Omega, \ \lambda_1, \dots, \lambda_k \in \Lambda_+, \sum_{i=1}^{k} \lambda_i = I_Y, \ k \in \mathbb{N} \right\}.$$

Theorem 6. *Let X be a vector space, F a Dedekind complete vector lattice and $\Lambda :=$ $\mathrm{Orth}(F)$. For a sublinear operator $P : X \to F$ the representation holds:*

$$\partial P = o\text{-}\mathrm{cl}(\mathrm{co}_\Lambda(\mathrm{ext}(\partial P))).$$

$\lhd$ This is an operator version of the classical Kreĭn–Mil′man theorem, obtained by A. G. Kusraev and S. S. Kutateladze in [5]; see also [9, § 2.4]. $\rhd$

4. Proofs and Corollaries

We now are able proceed to prove the above results.

PROOF OF THEOREM 1. If $S \in \mathrm{ext}\,\partial(T \circ P)$ then $S = T \circ R$ for some $R \in \partial(P)$ by Theorem 4. So, we just need to ensure that $R \in \mathrm{ext}\,\partial(P)$. Assume first that T is strictly positive. For $x \in X$ and $y \in F$ denote $v := \inf_{u \in X} v_{x,y}(u)$ where

$$v_{x,y}(u) := (P(u) - Ru) \vee (P(u - x) - R(u - x) + y) - y^+$$

and observe that $v \geqslant 0 \vee y - y^+ = 0$ as $P(u) \geqslant Ru$ and $P(u - x) \geqslant R(u - x)$. Moreover, $Tv \leqslant Tv_{x,y}(u)$ for all $u \in X$ and $y \in F$, so that $0 \leqslant Tv \leqslant \inf_{x \in X} Tv_{x,y}(u) = 0$ according to Theorem 5. It follows that $v = 0$ and, applying Theorem 5 again (this time with $T = I_F$), we arrive at the required inclusion $R \in \mathrm{ext}\,\partial(P)$.

In the general case consider the band projection π onto the carrier $\mathscr{C}_T$ of T defined as $\mathscr{C}_T := \{x \in E : T(|x|) = 0\}^\perp$, see [1, page 51]. Clearly, T is strictly positive on $\mathscr{C}_T$; therefore, applying what has already been proven to the operator $\pi \circ P : X \to \mathscr{C}_T$, we get $\mathrm{ext}\,\partial(T \circ \pi \circ P) \subset$ $T \circ \mathrm{ext}\,\partial(\pi \circ P)$. Thereby, $S = T \circ R$ for some $R \in \pi \circ \mathrm{ext}\,\partial(P)$, since $\mathrm{ext}(\pi \circ \partial(P)) = \pi \circ \mathrm{ext}\,\partial(P)$, see [5, 2.2.6(1)]. Take an arbitrary operator $R_0 \in \mathrm{ext}(\pi' \circ P)$ with $\pi' := I_F - \pi$, whose existence is guaranteed by Theorem 6. Considering that $R(X) \subset \mathscr{C}_T$ and $R_0(X) \subset \ker(\pi) = \pi'(F)$ we deduce

$$S = T \circ R = T \circ (\pi \circ R + \pi' \circ R_0) \in T \circ \big(\pi \circ \mathrm{ext}\,\partial(P) + \pi' \circ \mathrm{ext}\,\partial(P)\big) \subset T \circ \mathrm{ext}\,\partial(P),$$

where the last inclusion follows from the fact that the mixing of extreme operators is also an extreme operator, see [5, 2.2.8(1)]. $\rhd$

Corollary 1. *For every $R \in \partial(T \circ P)$ there exists a net (R_i) in $\mathrm{co}_\Lambda(\mathrm{ext}\,\partial(P))$ such that $T \circ R_i$ is point-wise order convergent to R.*

Lemma. *Let D be a majorizing subspace of a preordered vector space E and F a Dedekind complete vector lattice. For a positive operator $S : D \to F$ define the mapping $p_S : E \to F$ as*

$$p_S(x) := \inf\{Sx' : \ x' \in D, \, x \leqslant x'\} \quad (x \in E).$$

Then $p_S : E \to F$ *is a sublinear operator and* $\partial(p_S) = \mathscr{E}(S)$.

$\lhd$ This simple fact is often used in the theory of positive operators, see, for example, [1, Theorem 1.32], [9, Theorem 1.4.15 (1)] and [10, Remark 2]. $\rhd$

Proof of Theorem 2. Denote $U_S(x) := \{S(x') : x' \in D,\ x' \geqslant x\}$ and note that $U_{T\circ S}(x) = T(U_S(x))$ for all $x \in E$. Moreover, $U_S(x)$ is downward directed, since $y, z \in U_S(x)$ implies $y \wedge z \in U_S(x)$. These two facts together with the order continuity of T yield

$$p_{T\circ S}(x) = \inf U_{T\circ S}(x) = \inf T(U_S(x)) = T(\inf U_S(x)) = T(p_S(x)).$$

So, $p_{T\circ S} = T \circ p_S$ and, applying Theorems 4 and 1 together with the above lemma, we deduce the desired relations:

$$\mathscr{E}(T\circ S) = \partial(p_{T\circ S}) = \partial(T\circ p_S) = T\circ\partial(p_S) = T\circ\mathscr{E}(S);$$
$$\operatorname{ext}\mathscr{E}(T\circ S) = \operatorname{ext}\partial(p_{T\circ S}) = \operatorname{ext}\partial(T\circ p_S) \subset T\circ\operatorname{ext}\partial(p_S) = T\circ\operatorname{ext}\mathscr{E}(S). \ \rhd$$

Corollary 2. *For every* $R \in \mathscr{E}(T\circ S)$ *there exists a net* (R_i) *in* $\operatorname{co}_\Lambda(\operatorname{ext}\mathscr{E}(S))$ *such that* $T\circ R_i$ *is point-wise order convergent to* R.

Corollary 3. *If* S *is a lattice homomorphism, then each* $R \in \mathscr{E}(T\circ S)$ *is a point-wise o-limit of a net* $(T\circ R_i)$, *where* $R_i : E \to F$ *are* Λ*-convex combinations of lattice homomorphisms.*

Corollary 4. *Assume that* $h : H \to E$ *is a lattice homomorphism with* $h(H)$ *a majorizing sublattice of* E *and* $S \in L^+(h(H), F)$. *Denote by* $\mathscr{E}_h(S)$ *and* $\mathscr{E}_h(T\circ S)$ *the collections of positive operators* $U : E \to F$ *and* $V : E \to G$ *such that* $U\circ h = S\circ h$ *and* $V\circ h = T\circ S\circ h$, *respectively. Then the following relations hold:*

$$\mathscr{E}_h(T\circ S) = T\circ\mathscr{E}_h(S);$$
$$\operatorname{ext}\mathscr{E}_h(T\circ S) \subset T\circ\operatorname{ext}\mathscr{E}_h(S).$$

Proof of corollaries 1-4. Corollaries 1 and 2 are immediate from Theorems 1, 2, and 6. The third corollary follows from the second one taking into account the following result (known as the Lipecki–Luxemburg theorem): An operator $R \in \mathscr{E}(T)$ is an extreme point of $\mathscr{E}(T)$ if and only if R is a lattice homomorphism, [1, Theorem 2.51]. To verify Corollary 4, one only needs to apply Theorem 2 with $D = h(H)$.

Proof of Theorem 3. Given $y \in E$, one can take $x \in D$ with $|y| \leqslant x$ as D is a majorizing sublattice. Then for every $\theta \in \mathscr{P}_{\mathrm{fin}}(\mathrm{A})$ we have

$$\sum_{\alpha\in\theta}|\hat{S}_\alpha(y)| \leqslant \sum_{\alpha\in\theta}|\hat{S}_\alpha(x)| = \sum_{\alpha\in\theta}|S_\alpha(x)| \leqslant S(x),$$

hence the family $(\hat{S}_\alpha y)_{\alpha\in\mathrm{A}}$ is order summable.

Denote by F, Σ, and $\mathbb{S}$ respectively the set of all order summable families in G indexed by A, the summation operator from F to G, and an operator from D to F whose α-th components are S_α; in symbols,

$$F := \Big\{(g_\alpha)_{\alpha\in\mathrm{A}} \in G^{\mathrm{A}} : o\text{-}\sum\nolimits_{\alpha\in\mathrm{A}}|g_\alpha| \in G\Big\},$$
$$\Sigma u := o\text{-}\sum_{\alpha\in\mathrm{A}} g_\alpha \quad (u := (g_\alpha)_{\alpha\in\mathrm{A}} \in F),$$
$$\mathbb{S}x := (S_\alpha x)_{\alpha\in\mathrm{A}} \in F \quad (x \in D).$$

Then F is a Dedekind complete vector lattice under component-wise addition, scalar multiplication, and ordering, whilst Σ is a strictly positive Maharam operator and $\mathbb{S}$ is a positive operator. By Theorem 2, $\mathscr{E}(\Sigma \circ \mathbb{S}) = \Sigma \circ \mathscr{E}(\mathbb{S})$ and $\operatorname{ext} \mathscr{E}(\Sigma \circ \mathbb{S}) \subset \Sigma \circ \operatorname{ext} \mathscr{E}(\mathbb{S})$, from which the required follows. $\triangleright$

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КРАЙНИЕ ПРОДОЛЖЕНИЯ ПОЛОЖИТЕЛЬНЫХ ОПЕРАТОРОВ

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Аннотация. Рассматриваются векторные решетки E и F и положительный оператор S из мажорирующего подпространства $D \subset E$ в F. Символом $\mathscr{E}(S)$ обозначается множество всех положительных продолжений оператора S на всю решетку E. Цель настоящей заметки — описание крайних точек множества $\mathscr{E}(T \circ S)$. Установлено, в частности, что выпуклые множества $\mathscr{E}(T \circ S)$ и $T \circ \mathscr{E}(S)$ совпадают и каждая крайняя точка $\mathscr{E}(T \circ S)$ является крайней точкой $T \circ \mathscr{E}(S)$, если $T : F \to G$ оператор Магарам между порядково полными векторными решетками. Доказательство опирается на следующие три известных факта: характеризация крайних точек субдифференциала (и, тем самым, крайних продолжений положительного оператора), абстрактное дезинтегрирование в пространствах Канторовича и внутренняя характеризация опорных множеств сублинейных операторов.

Ключевые слова: векторная решетка, положительный оператор, крайнее продолжение, оператор Магарам, субдифференциал, абстрактное дезинтегрирование.

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