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AUTOMATIC BOUNDEDNESS OF SOME OPERATORS
BETWEEN ORDERED AND TOPOLOGICAL VECTOR SPACES[#]

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To blessed memory of my wise friend and teacher Semën Kutateladze

Abstract. Order-to-topology continuous operators and order-to-norm bounded operators have been recently studied by many authors mostly in the framework of Banach lattices. In the present note, we extend some of results obtained by these authors to the setting of operators from an ordered Banach space to a topological vector space. We present several conditions providing topological boundedness of such operators, and investigate uniform boundedness principle for collectively qualified families of operators, and establish uniform boundedness of power order-to-norm bounded operator semigroups on an ordered Banach space with a closed generating cone. We prove that every collectively order-to-topology bounded set of operators from an ordered Banach space to a topological vector space is collective ru-to-topology continuous and provide conditions under which such sets are uniformly bounded.

Keywords: ordered vector space, topological vector space, ordered Banach space, order-to-topology bounded operator, order-to-topology continuous operator.

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1. Introduction

Order-to-topology continuous and order-to-norm bounded operators have been studied recently by different authors [1–7]. In the present note, some of their results are extended to the setting of topological vector spaces and new conditions for automatic boundedness of operators are given. We abbreviate normed, topological, ordered, ordered normed, and ordered Banach vector spaces as NS, TVS, OVS, ONS, and OBS, respectively. In what follows, vector spaces are real, operators are linear, symbol $\mathcal{L}(X, Y)$ stands for the space of operators from a vector space X to a vector space Y , B_X for the closed unit ball of a NS X , and $x_\alpha \downarrow 0$ for a decreasing net in an OVS such that $\inf_\alpha x_\alpha = 0$. A net (x_α) in an OVS X

– *order converges* to x (*o-converges* to x , or $x_\alpha \xrightarrow{o} x$) if there exists a net $g_\beta \downarrow 0$ in X such that, for each β there is α_β such that $\pm(x_\alpha - x) \leq g_\beta$ for $\alpha \geq \alpha_\beta$;

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– *relative uniform converges* to x (*ru-converges* to x , or $x_\alpha \xrightarrow{ru} x$) if, for some $u \in X_+$ there exists an increasing sequence (α_n) of indices with $\pm(x_\alpha - x) \leq \frac{1}{n}u$ for $\alpha \geq \alpha_n$.

We need the following classes of operators.

DEFINITION 1.1. An operator T between OVSs X and Y is

a) *order bounded* if $T[a, b]$ is order bounded for every order interval $[a, b]$ in X (the set of such operators will be denoted by $\mathcal{L}_{ob}(X, Y)$).

b) *ru-continuous* if $Tx_\alpha \xrightarrow{ru} 0$ in Y whenever $x_\alpha \xrightarrow{ru} 0$ (shortly, $T \in \mathcal{L}_{rc}(X, Y)$).

An operator T from OVS X to TVS (Y, τ) is

c) *order-to-topology continuous* if $Tx_\alpha \xrightarrow{\tau} 0$ whenever $x_\alpha \xrightarrow{o} 0$ (shortly, $T \in \mathcal{L}_{orc}(X, Y)$).

If Y is a NS, the set of such operators is denoted by $\mathcal{L}_{onc}(X, Y)$.

d) *ru-to-topology continuous* if $Tx_\alpha \xrightarrow{\tau} 0$ whenever $x_\alpha \xrightarrow{ru} 0$ (shortly, $T \in \mathcal{L}_{rtc}(X, Y)$).

If Y is a NS, the set of such operators is denoted by $\mathcal{L}_{rnc}(X, Y)$.

e) *order-to-topology bounded* if $T[a, b]$ is τ -bounded for every $[a, b] \subseteq X$ (shortly, $T \in \mathcal{L}_{orb}(X, Y)$). If Y is a NS, the set of such operators is denoted by $\mathcal{L}_{omb}(X, Y)$.

Clearly, all sets of operators mentioned in Definition 1.1 are vector spaces. We also recall several definitions concerning collective convergences and collectively qualified sets of operators (see, [2–4; 8–10]). For convenience, we say that a family $\mathcal{B} = \{(x_\alpha^b)_{\alpha \in A}\}_{b \in B}$ of nets in a TVS $X = (X, \tau)$ indexed by the same directed set A *collective τ -converges* to 0 (briefly, $\mathcal{B} \xrightarrow{c-\tau} 0$) whenever, for each $U \in \tau(0)$ there exists α_U with $x_\alpha^b \in U$ for all $\alpha \geq \alpha_U$ and $b \in B$.

DEFINITION 1.2. A set \mathcal{T} of operators between OVSs X and Y is

a) *collectively order bounded* ($\mathcal{T} \in \mathbf{L}_{ob}(X, Y)$), if $\mathcal{T}[a, b]$ is order bounded for every $[a, b] \subseteq X$.

A set \mathcal{T} of operators from an OVS X to a TVS (Y, τ) is

b) *collectively order-to-topology continuous* ($\mathcal{T} \in \mathbf{L}_{orc}(X, Y)$), if $\mathcal{T}x_\alpha \xrightarrow{c-\tau} 0$ whenever $x_\alpha \xrightarrow{o} 0$. If additionally Y is a NS, we write $\mathcal{T} \in \mathbf{L}_{onc}(X, Y)$.

c) *collectively ru-to-topology continuous* ($\mathcal{T} \in \mathbf{L}_{rtc}(X, Y)$), if $\mathcal{T}x_\alpha \xrightarrow{c-\tau} 0$ whenever $x_\alpha \xrightarrow{ru} 0$. If additionally Y is a NS, we write $\mathcal{T} \in \mathbf{L}_{rnc}(X, Y)$.

d) *collectively order-to-topology bounded* ($\mathcal{T} \in \mathbf{L}_{orb}(X, Y)$), if $\mathcal{T}[a, b]$ is τ -bounded for every $[a, b] \subseteq X$. If additionally Y is a NS, we write $\mathcal{T} \in \mathbf{L}_{omb}(X, Y)$.

A set \mathcal{T} of operators from a TVS (X, ξ) to a TVS (Y, τ) is

e) *collectively continuous* ($\mathcal{T} \in \mathbf{L}_c(X, Y)$), if $\mathcal{T}x_\alpha \xrightarrow{c-\tau} 0$ whenever $x_\alpha \xrightarrow{\xi} 0$.

f) *collectively bounded* ($\mathcal{T} \in \mathbf{L}_b(X, Y)$), if $\mathcal{T}U$ is τ -bounded whenever U is ξ -bounded.

The σ -versions of Definitions 1.1 and 1.2 are obtained via replacing nets by sequences, and the corresponding spaces (classes) are denoted by $\mathcal{L}_{orc}^\sigma(X, Y)$ ($\mathbf{L}_{orc}^\sigma(X, Y)$), etc. Let X and Y be OVSs. Then $r_1\mathcal{T}_1 + r_2\mathcal{T}_2, \mathcal{T}_1 \cup \mathcal{T}_2 \in \mathbf{L}_{ob}(X, Y)$ for every $r_1, r_2 \in \mathbb{R}$ and nonempty subsets $\mathcal{T}_1, \mathcal{T}_2$ of $\mathbf{L}_{ob}(X, Y)$. The same is true for $\mathbf{L}_{orc}(X, Y), \mathbf{L}_{rtc}(X, Y), \mathbf{L}_{rnc}(X, Y)$, etc.

The present note is organized as follows. Theorem 2.1 asserts collective ru-to-topology continuity of collectively order-to-topology bounded sets. In Theorem 2.2, conditions for the inclusion $\mathbf{L}_{orc}(X, Y) \subseteq \mathbf{L}_{orb}(X, Y)$ are given. Theorem 2.3 gives conditions for the inclusion $\mathbf{L}_{orb}(X, Y) \subseteq \mathbf{L}_b(X, Y)$.

For the terminology and notations that are not explained in the text, we refer to [11, 12].

2. Main Results

We start with the following theorem which tells us that collectively order-to-topology bounded sets quite often agree with collectively ru-to-topology continuous sets.

Theorem 2.1. *Let X be an OVS and (Y, τ) a TVS. Then $\mathbf{L}_{orb}(X, Y) \subseteq \mathbf{L}_{r\tau c}(X, Y)$. If additionally X_+ is generating then $\mathbf{L}_{orb}(X, Y) = \mathbf{L}_{r\tau c}(X, Y)$.*

\triangleleft If, on the contrary, $\mathcal{F} \in \mathbf{L}_{orb}(X, Y) \setminus \mathbf{L}_{r\tau c}(X, Y)$ then, for some $x_\alpha \xrightarrow{ru} 0$ there exists an absorbing $U \in \tau(0)$ such that, for every α there exist $\alpha' \geq \alpha$ and $T_\alpha \in \mathcal{F}$ with $T_\alpha x_{\alpha'} \notin U$.

Since $x_\alpha \xrightarrow{ru} 0$, for some $u \in X_+$ there exists an increasing sequence (α_n) of indices with $\pm n x_\alpha \leq u$ for $\alpha \geq \alpha_n$. It follows from $\mathcal{F} \in \mathbf{L}_{orb}(X, Y)$ that $\mathcal{F}[-u, u] \subseteq NU$ for some $N \in \mathbb{N}$. Since $n x_\alpha \in [-u, u]$ for $\alpha \geq \alpha_n$ and $[\alpha_n]' \geq \alpha_n$ then $n x_{[\alpha_n]'} \in [-u, u]$, and hence $T_{\alpha_n}(n x_{[\alpha_n]'}) \in NU$ for every n . In particular, $T_{\alpha_n}(x_{[\alpha_n]'}) \in U$, which is absurd. We conclude $\mathbf{L}_{orb}(X, Y) \subseteq \mathbf{L}_{r\tau c}(X, Y)$.

Now, suppose X_+ is generating, and let $\mathcal{F} \in \mathbf{L}_{r\tau c}(X, Y) \setminus \mathbf{L}_{orb}(X, Y)$. Since $X = X_+ - X_+$ and $\mathcal{F} \notin \mathbf{L}_{orb}(X, Y)$, there exist $x \in X_+$ and an absorbing $U \in \tau(0)$ with $\mathcal{F}[-x, x] \not\subseteq nU$ for every $n \in \mathbb{N}$. Therefore, $T_n x_n \notin nU$ for all n and some sequences (x_n) in $[-x, x]$ and (T_n) in \mathcal{F} . Since $\frac{1}{n} x_n \xrightarrow{ru} 0$ and $\mathcal{F} \in \mathbf{L}_{r\tau c}(X, Y)$, there exists a sequence (n_k) such that $T(\frac{1}{n} x_n) \in U$ for all $n \geq n_k$ and $T \in \mathcal{F}$. Then $T_{n_1} x_{n_1} \in n_1 U$, which is a contradiction. So, $\mathbf{L}_{r\tau c}(X, Y) \subseteq \mathbf{L}_{orb}(X, Y)$, and hence $\mathbf{L}_{orb}(X, Y) = \mathbf{L}_{r\tau c}(X, Y)$. \triangleright

Corollary 2.1. *Every order-to-topology bounded operator from an OVS X to a TVS (Y, τ) is ru -to-topology continuous. Assuming in addition that X_+ is generating, $\mathcal{L}_{orb}(X, Y) = \mathcal{L}_{r\tau c}(X, Y)$.*

The following result can be viewed as a topological version of [4, Theorem 2.1] (in the vector lattice setting see also [2, Theorem 2.1]), [11, Lemma 1.72] and [13, Theorem 2.1].

Theorem 2.2. *Let X be an Archimedean OVS with a generating cone and (Y, τ) a TVS. Then $\mathbf{L}_{orc}(X, Y) \subseteq \mathbf{L}_{orb}(X, Y)$.*

\triangleleft We argue to a contradiction supposing $\mathcal{F} \in \mathbf{L}_{orc}(X, Y) \setminus \mathbf{L}_{orb}(X, Y)$. Theorem 2.1 implies $\mathcal{F} \notin \mathbf{L}_{r\tau c}(X, Y)$. So, $\mathcal{F} x_\alpha \not\xrightarrow{c-\tau} 0$ for some net (x_α) in X such that $x_\alpha \xrightarrow{ru} 0$. Since X is Archimedean then $x_\alpha \xrightarrow{o} 0$, and hence $\mathcal{F} x_\alpha \xrightarrow{c-\tau} 0$ because of $\mathcal{F} \in \mathbf{L}_{orc}(X, Y)$. The obtained contradiction completes the proof. \triangleright

Corollary 2.2. *Every order-to-topology continuous operator from an Archimedean OVS with a generating cone to a TVS is order-to-topology bounded.*

It is well known that $\mathbf{L}_c(X, Y) = \mathbf{L}_b(X, Y)$ whenever X and Y are NSs. The following result can be viewed as a partial extension of [2, Theorem 2.1] and [4, Theorem 2.8].

Theorem 2.3. *Let X be an OBS with a closed generating cone and (Y, τ) a TVS. Then $\mathbf{L}_{orb}(X, Y) \subseteq \mathbf{L}_b(X, Y)$.*

\triangleleft Let $\mathcal{F} \in \mathbf{L}_{orb}(X, Y)$. Suppose, on the contrary, $\mathcal{F} \notin \mathbf{L}_b(X, Y)$. By the Krein–Smulian theorem (cf. [12, Theorem 2.37]), $\alpha B_X \subseteq B_X \cap X_+ - B_X \cap X_+$ for some $\alpha > 0$, and hence $\mathcal{F}(B_X \cap X_+)$ is not τ -bounded. Then, there exists an absorbing $U \in \tau(0)$ with $\mathcal{F}(B_X \cap X_+) \not\subseteq nU$ for every $n \in \mathbb{N}$. So, for some sequences (x_n) in $B_X \cap X_+$ and (T_n) in \mathcal{F} we have $T_n x_n \notin n^3 U$ for all n . Set

$$x := \|\cdot\| - \sum_{n=1}^{\infty} n^{-2} x_n \in X_+.$$

Since $\mathcal{F} \in \mathbf{L}_{orb}(X, Y)$ then $\mathcal{F}[0, x] \subseteq NU$ for some $N \in \mathbb{N}$. It follows from $n^{-2} x_n \in [0, x]$ that $T_n(n^{-2} x_n) \in NU \subseteq nU$ for large enough n . This is absurd, because $T_n(n^{-2} x_n) \notin nU$ for all n . \triangleright

Norm completeness of X is essential in Theorem 2.3. For example, an operator $T \in \mathcal{L}(c_{00})$ defined by $Tx = (\sum_{k=1}^{\infty} x_k) e_1$ is order-to-norm bounded, yet not bounded. To see that the condition $X_+ - X_+ = X$ is essential, it is enough to take any unbounded operator T

on a Banach space X with a trivial cone $X_+ = \{0\}$. The closeness of X_+ in a Banach space X is also essential [4, Example 2.12 b)]. The next corollary is a kind of Uniform Boundedness Principle for families of operators which need not be continuous a priori.

Corollary 2.3. *Every collectively order-to-norm bounded set of operators from an OBS with a closed generating cone to a NS is uniformly bounded.*

Corollary 2.4. *Every order-to-norm bounded operator from an OBS with a closed generating cone to a NS is bounded.*

The following corollary of Theorem 2.3 provides an automatic continuity result that extends the well known fact (see, for example, [12, Theorem 2.32]) that every positive operator from an OBS with a closed generating cone to an OBS with a closed cone is continuous.

Corollary 2.5. *Every order bounded operator from an OBS with a closed generating cone to an ONS with a normal cone is continuous.*

Since in NSs bounded linear operators are continuous, and since weak compact sets are bounded in Banach spaces, we obtain the following consequence of Theorem 2.3.

Corollary 2.6. *Every operator from an OBS with a closed generating cone to a Banach space is continuous whenever it takes order intervals onto relatively weak compact sets.*

Collective boundedness of a semigroup generated by a single operator is known as power boundedness of the operator. We say that an operator T on an ordered TVS (X, τ) is power order-to-topology bounded if the set $\bigcup_{n=1}^{\infty} T^n[a, b]$ is τ -bounded for every $[a, b]$ in X . An operator semigroup \mathcal{S} on an ordered TVS (X, τ) is order-to-topology bounded if the set $\bigcup_{T \in \mathcal{S}} T[a, b]$ is τ -bounded for every $[a, b]$ in X . The next two corollaries deal with these notions.

Corollary 2.7. *Every power order-to-norm bounded operator on an OBS with a closed generating cone is power bounded.*

Corollary 2.8. *Every power order-to-norm bounded operator semigroup on an OBS with a closed generating cone is uniformly continuous.*

Since $\mathbf{L}_b(X, Y) \subseteq \mathbf{L}_{ob}(X, Y)$ whenever X is a normal OVS and Y is a NS, the next collective extension of [3, Proposition 1.5] follows from Theorems 2.1 and 2.3.

Proposition 2.1. *Let X be an OBS with a closed generating normal cone and Y a NS. Then $\mathbf{L}_{rnc}(X, Y) = \mathbf{L}_{ob}(X, Y) = \mathbf{L}_b(X, Y)$.*

Corollary 2.9. *Let X be an OBS with a closed generating normal cone and Y a NS. Then $\mathcal{L}_{rnc}(X, Y) = \mathcal{L}_{ob}(X, Y) = \mathcal{L}_b(X, Y)$.*

By [4, Theorem 2.4], $\mathcal{L}_{rc}(X, Y) = \mathcal{L}_{ob}(X, Y)$ whenever X and Y are OVSs with generating cones. Therefore, we have one more consequence of Proposition 2.1 that provides conditions for automatic boundedness of ru-continuous operators.

Corollary 2.10. *Let X be an OBS with a closed generating normal cone and Y an OVS with a generating normal cone. Then $\mathcal{L}_{rc}(X, Y) \subseteq \mathcal{L}_b(X, Y)$.*

Proposition 2.2 [4, Proposition 2.11]. *Let X and Y be OBSs with closed generating cones, and $X^\sim \neq \emptyset$. Then every $\mathcal{T} \in \mathbf{L}_{ob}(X, Y)$ is uniformly bounded iff Y_+ is normal.*

◁ If Y_+ is not normal, then the interval $[0, y_0]$ is not bounded for some $y_0 \in X_+$. Pick $f_0 \neq 0$ in X^\sim and take $x_0 \in X$ with $f_0(x_0) = 1$. Then, $\{f_0 \otimes y\}_{y \in [0, y_0]} \in \mathbf{L}_{ob}(X, Y)$. However, the set $\{f_0 \otimes y\}_{y \in [0, y_0]}$ is not uniformly bounded since

$$\bigcup_{y \in [0, y_0]} (f_0 \otimes y)(x_0) = [0, y_0]$$

is not bounded.

If Y_+ is normal, then $\mathbf{L}_{ob}(X, Y) \subseteq \mathbf{L}_{omb}(X, Y)$ by [4, Proposition 2.6]. The rest follows from Proposition 2.1. \triangleright

The following proposition is an extension of [3, Lemma 2.1].

Proposition 2.3. *Let X be a normal OBS and (Y, τ) a Banach space with a dual topology. Then $\mathcal{L}_{\sigma_{rc}}^\sigma(X, Y) \subseteq \mathcal{L}_{omb}(X, Y)$.*

\triangleleft On the way to contradiction, assume $T \in \mathcal{L}_{\sigma_{rc}}^\sigma(X, Y) \setminus \mathcal{L}_{omb}(X, Y)$. Then $T[0, u]$ is not bounded for some $u \in X_+$. Since X is normal, $[0, u]$ is bounded, say $\sup_{x \in [0, u]} \|x\| \leq M$. Pick a sequence (u_n) in $[0, u]$ with $\|Tu_n\| \geq n2^n$, and set

$$y_n := \|\cdot\| - \sum_{k=n}^{\infty} 2^{-k} u_k \quad \text{for } n \in \mathbb{N}.$$

Then $y_n \downarrow \geq 0$. Let $0 \leq y_0 \leq y_n$ for all $n \in \mathbb{N}$. Since $0 \leq y_0 \leq 2^{1-n}u$ and $\|2^{1-n}u\| \leq M2^{1-n} \rightarrow 0$ then $y_0 = 0$ by [12, Theorem 2.23]. So, $y_n \downarrow 0$, and hence $y_n \xrightarrow{o} 0$. We deduce from $T \in \mathcal{L}_{\sigma_{rc}}^\sigma(X, Y)$ that $Ty_n \xrightarrow{\tau} 0$. Since the topology τ is dual then $Ty_n \xrightarrow{w} 0$. Therefore, the sequence (Tny_n) is norm bounded. This is absurd because

$$\|Ty_{n+1} - Ty_n\| = \|T(2^{-n}u_n)\| \geq n \quad (\forall n \in \mathbb{N}). \quad \triangleright$$

Corollary 2.11. *Every order-to-topology σ -continuous operator from a normal OBS to a Banach space with a dual topology is order-to-norm bounded.*

Combining Corollaries 2.11 and 2.4, gives the last result of our note.

Corollary 2.12. *Every order-to-topology σ -continuous operator from an OBS with a closed generating normal cone to a Banach space with a dual topology is bounded.*

References

1. Alpay, S., Emelyanov, E. and Gorokhova, S. σ -Continuous, Lebesgue, KB, and Levi Operators Between Vector Lattices and Topological Vector Spaces, *Results in Mathematics*, 2022, vol. 77, article no. 117. DOI: 10.1007/s00025-022-01650-3.
2. Emelyanov, E. Collective Order Convergence and Collectively Qualified Sets of Operators, *Siberian Electronic Mathematical Reports*, arxiv.org/pdf/2408.03671 (to appear).
3. Emelyanov, E. *On Automatic Boundedness of Some Operators in Ordered Banach Spaces*, arxiv.org/pdf/2503.18834.
4. Emelyanov, E., Erkursun-Özcan, N. and Gorokhova, S. Collective Order Boundedness of Sets of Operators Between Ordered Vector Spaces, *Results in Mathematics*, 2025, vol. 80, article no. 70. DOI: 10.1007/s00025-025-02386-6.
5. Jalili, S. A., Azar, K. Y. and Moghimi, M. B. F. Order-to-Topology Continuous Operators, *Positivity*, 2021, vol. 25, pp. 1313–1322. DOI: 10.1007/s11117-021-00817-6.
6. Keleş, C. S., Turan, B. and Altın, B. Order Structure of Order-to-Topological Continuous Operator, *Turkish Journal of Mathematics*, 2025, vol. 49 (2), pp. 173–184. DOI: 10.55730/1300-0098.3581.
7. Zhang, F., Shen, H. and Chen, Z. Property (h) of Banach Lattice and Order-to-Norm Continuous Operators, *Mathematics*, 2023, vol. 11, 2747. DOI: 10.3390/math11122747.
8. Alpay, S., Emelyanov, E. and Gorokhova, S. *On Collectively Almost (Limitedly, Order) L-Weakly Compact Sets of Operators*, arxiv.org/pdf/2407.11885.
9. Emelyanov, E. On Collectively σ -Levi Sets of Operators, *Vladikavkaz Mathematical Journal*, 2025, vol. 27, no. 1, pp. 36–43. DOI: 10.46698/y6929-3405-2251-o.
10. Emelyanov, E. On Collectively L-Weakly Compact Sets of Operators, *Analysis Mathematica*, 2025, vol. 51 (2), pp. 447–455. DOI: 10.1007/s10476-025-00088-3.
11. Aliprantis, C. D. and Burkinshaw, O. *Locally Solid Riesz Spaces with Applications to Economics*, 2nd edition, Providence, American Mathematical Society, 2003.
12. Aliprantis, C. D. and Tourky, R. *Cones and Duality*, Providence, American Mathematical Society, 2007.
13. Abramovich, Y. and Sirotkin, G. On Order Convergence of Nets, *Positivity*, 2005, vol. 9, pp. 287–292. DOI: 10.1007/s11117-004-7543-x.

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К АВТОМАТИЧЕСКОЙ ОГРАНИЧЕННОСТИ НЕКОТОРЫХ ОПЕРАТОРОВ
МЕЖДУ УПОРЯДОЧЕННЫМИ И ТОПОЛОГИЧЕСКИМИ
ВЕКТОРНЫМИ ПРОСТРАНСТВАМИ

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Аннотация. Порядково-топологически непрерывные операторы и операторы, переводящие порядковые интервалы в ограниченные по норме множества, изучались в последние годы многими авторами, в основном, в контексте банаховых решеток. В настоящей заметке некоторые из результатов, полученные этими авторами, распространяются на случай операторов, действующих из упорядоченного банахова пространства в топологическое векторное пространство. Устанавливается несколько условий, обеспечивающих топологическую ограниченность вышеупомянутых операторов, и исследуется принцип равномерной ограниченности для различных совместно квалифицированных семейств операторов. Среди прочего, доказывается равномерная ограниченность операторных полугрупп, ограниченных по норме на всяком порядковом интервале и действующих в упорядоченном банаховом пространстве с замкнутым порождающим конусом. В заметке также устанавливается, что каждое совместно порядково-топологически ограниченное множество операторов из упорядоченного банахова пространства в топологическое векторное пространство будет коллективно ru -топологически непрерывным, и приводятся условия, обеспечивающие равномерную ограниченность таких множеств.

Ключевые слова: упорядоченное векторное пространство, топологическое векторное пространство, упорядоченное банахово пространство, порядково-топологически ограниченный оператор, порядково-топологически непрерывный оператор.

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